Math 200, Nov 30

Today: (2009 WT 2) ∩ (Chapter 15)
Wed: " " 1 or 2 ∩ Chapter 14
Frid: " 1 or 2 ∩ Chapter 12

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2009 WT 2:

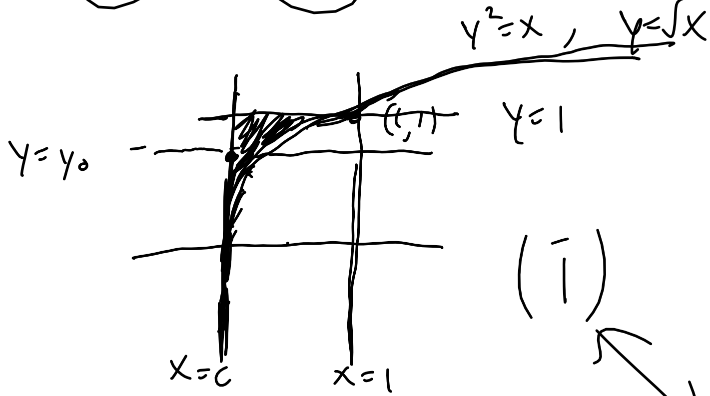
Problem 6: Consider

$$\int_0^1 \left(\int_{\sqrt{x}}^1 \sqrt{1+y^3}\, dy \right) dx$$

(i) Sketch the region of integration

(ii) Evaluate

$$\int_{x=0}^{x=1} \left(\int_{y=\sqrt{x}}^{y=1} \sqrt{1+y^3}\, dy \right) dx$$



for $0 \le x \le 1$,

fix x, y varies:

$\sqrt{x} \le y \le 1$

$y=\sqrt{x}$, $y^2=x$

fix $y=y_0$ $\quad 0 \le y = y_0 \le 1$

x vary: $0 \le x \le y^2$

$(\sqrt{x} \le y \Leftrightarrow x \le y^2)$

$$\int_{y=0}^{y=1} \underbrace{\left(\int_{x=0}^{x=y^2} \sqrt{1+y^3}\, dx \right)}_{x \text{ varying} \quad y \text{ constant}} dy$$

$$\int_{y=0}^{y=1} \left(\sqrt{1+y^3}\ \underbrace{x \Big|_{x=0}^{x=y^2}}\ \right) dy$$

$$\int_{y=0}^{y=1} \left(\sqrt{1+y^3}\right)\left(y^2 - 0 \right) dy$$

$$\int_{y=0}^{y=1} \sqrt{1+y^3}\ y^2\, dy$$

=

$f(y^2)' = (f')(y^2)(2y)$

$$\int \sqrt{1+y^3}\, y^2\, dy$$

$$= \left(\frac{1}{3}\right) \int \sqrt{1+y^3}\, \boxed{(3y^2)\, dy}$$

$$= \frac{1}{3} \frac{(1+y^3)^{3/2}}{3/2}$$

$=$

$$\frac{d}{dy} f(g(y)) = (f'(g(y)))(g'(y))$$

$$\frac{d}{dy}(f(y^3)) = (f'(y^3))\, 3y^2$$

$$\int e^{y^2}\, dy$$

$$\int e^{y^2} (2y)\, dy$$

$=$

Also

$$(f(y^3))' = ((f')(y^3))\, 3y^2$$

$$\int \sqrt{1+t}\, dt = \int (1+t)^{1/2}\, dt$$

$$= \frac{(1+t)^{3/2}}{3/2} + C$$

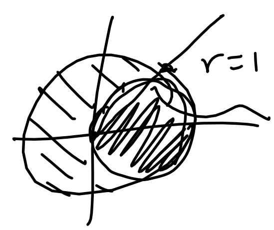
$$= \int_{y=0}^{y=1} \left(\sqrt{1+y^3}\; y^2\right) dy$$

$$= \frac{1}{3} \int_{y=0}^{y=1} \left(\sqrt{1+y^3}\right) (3y^2\, dy)$$

$$= \frac{1}{3} \left. \frac{(1+y^3)^{3/2}}{3/2} \right|_{y=0}^{y=1}$$

$$= \frac{1}{3} \frac{1}{3/2} \left(2^{3/2} - 1^{3/2}\right)$$

$=$ etc.



$=$

$$I = \int_{x=0}^{x=1} \left(\int_{y=\sqrt{x}}^{y=1} f(x,y)\, dy \right) dx$$

$$\sqrt{1+y^3}$$

$$1 \geq y \geq \sqrt{x}$$

$$y^2 \geq x$$

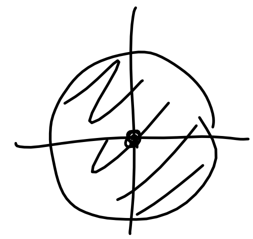
$$= \int_{y=0}^{y=1} \left(\int_{x=0}^{x=y^2} \sqrt{1+y^3}\, dx \right) dy$$

$$\bar{x} = \frac{\iint k \cdot x \, dA}{\text{Mass}}$$

$$= \frac{\int_{\vartheta=0}^{2\pi}\int_{r=0}^{r=1} k \cdot r\cos\vartheta \; r\,dr\,d\vartheta}{\int_{\vartheta=0}^{2\pi}\int_{r=0}^{r=1} k \; r\,dr\,d\vartheta}$$

= integrate in either order

7. Thin plate, uniform density, k, bounded by circle $x^2+y^2=1$. Find centre of mass

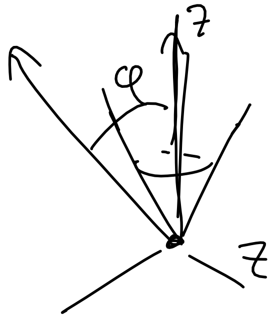


uniform density k

find $(\bar{x}, \bar{y})$

Polar: $0 \le r \le 1$
$0 \le \theta \le 2\pi$

$$\text{Mass} = \iint_{\text{Circle}} k \, dA = \int_{\theta=0}^{2\pi}\int_{r=0}^{1} k \cdot r\,dr\,d\theta$$



(φ = const)

$$z = \sqrt{3x^2+3y^2}$$
$$= \sqrt{3}\,\sqrt{x^2+y^2}$$
$$= \sqrt{3}\cdot r$$

r = polar radius (x,y)

Spherical: $z = \rho\cos\varphi$
$r = \rho\sin\varphi$

$$\rho = \sqrt{x^2+y^2+z^2}$$

(8) Let

$$I = \iiint_T (x^2+y^2)\, dV$$

T is solid bounded <u>below</u> by cone $z = \sqrt{3x^2+3y^2}$

<u>above</u> by $x^2+y^2+z^2=9$

$$\underline{\sqrt{3x^2+3y^2}} \le z \le \underline{\sqrt{9-x^2-y^2}}$$

(i) Cylindrical (ii) Spherical
(iii) Evaluate

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So θ = anything from 0, 2π

$$0 \le r \le 3/2$$

$$\int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=3/2} \left(\int_{\sqrt{3}\, r}^{\sqrt{9-r^2}} r^2 \, dz \right) r\,dr\,d\theta$$

$$\int_{\theta=0}^{2\pi} \int_{r=0}^{r=3/2} r^2 \left(\sqrt{9-r^2} - \sqrt{3}\, r\right) (r\,dr) \; d\theta$$

$$\left(r^2 \sqrt{9-r^2}\right) r\,dr \quad r^4\, dr$$

Cylindrical

$$\sqrt{3}\, r \le z \le \sqrt{9-r^2}$$

Limits are when

$$\sqrt{3}\, r = \sqrt{9-r^2}$$

or =

$$\sqrt{3}\sqrt{x^2+y^2} = \sqrt{9-x^2-y^2}$$

$$3r^2 = 9 - r^2$$

$$4r^2 = 9$$

$$2r = 3, \quad r = 3/2$$