Math 200, Nov 27

From now $\rightarrow$ end of classes
old final exam problems.

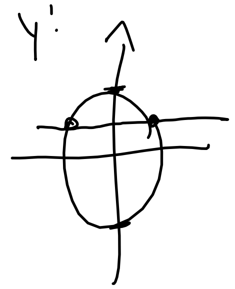
We know:

circle x, y plane

$x^2+y^2 \le 1$ or $0 \le r \le 1$, $0 \le \theta \le 2\pi$

$$\int \int_{x=\text{lower}(y)}^{x=\text{upper}(y)} dx\, dy$$

Fix y:



$x^2 \le 1-y^2$

$-\sqrt{1-y^2} \le x \le \sqrt{1-y^2}$

unit circle:

$y=1$ to $y=-1$ $\left(x = \sqrt{1-y^2} \text{ to } x = -\sqrt{1-y^2} \right)$

Problem 16, final 2012WT1

$$\int_{x=-1}^{x=1} \left(\int_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} \left(\int_{z=1-\sqrt{1-x^2-y^2}}^{z=1+\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)^{5/2}\, dz \right) dy \right) dx$$

Evaluate via spherical coordinates

$(x^2+y^2+z^2)^{5/2} = (\rho^2)^{5/2} = \rho^5$

$\rho = \sqrt{x^2+y^2+z^2}$

$-1 \le x \le 1$

$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

$y^2 \le \left(\sqrt{1-x^2}\right)^2$

$= 1-x^2$

$\rho^2 \sin^2\varphi = x^2+y^2 \le 1$

(spherical coords: z, $r = x^2+y^2$

ρ, φ: $\boxed{\begin{matrix} z = \rho\cos\varphi \\ r = \rho \sin\varphi \end{matrix}}$

z constraint:

$$0 \le \rho \le 2\cos\varphi$$

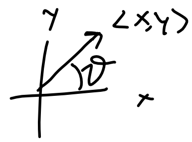
Other two $\Leftrightarrow$

$$x^2+y^2 \le 1$$

$$r^2 \le 1, \quad r \le 1$$

$$\rho \sin\varphi \le 1$$

ϑ



ϑ unconstrained in $[0, 2\pi]$

$$1-\underbrace{\sqrt{1-x^2-y^2}} \le z \le 1+\underbrace{\sqrt{1-x^2-y^2}}$$

$$-\sqrt{\underbrace{1-x^2-y^2}_{1-r^2}} \le z-1 \le +\sqrt{\underbrace{1-x^2-y^2}_{1-r^2}}$$

$$(z-1)^2 \le \left(\sqrt{1-x^2-y^2}\right)^2$$

$$z^2-2z+1 \le 1-x^2-y^2$$

$$\underbrace{x^2+y^2+z^2} \le 2z$$

$$\rho^2 \le 2\rho\cos\varphi$$

$\cos 0 = 1$ ← $\varphi = 0$

$\cos 90^\circ = 0$ → ← $\varphi = 90^\circ = \pi/2$

$\cos = -1$ ← $\varphi = 180^\circ = \pi$

$$0 \le \rho \le \min\left(\underset{\downarrow}{2\cos\varphi}\ ,\quad \right)$$

φ goes only from $0 \to 90^\circ$

$2\cos\varphi$ is $\underset{\varphi=0}{2} \to \underset{\varphi=45^\circ}{\sqrt{2}} \to \underset{\varphi=90^\circ}{0}$

$$\iiint f(x,y,z)\ \overset{\rho^2 \sin\varphi}{\swarrow}\ d\rho\, d\vartheta\, d\varphi$$

ϑ anything in $[0, 2\pi]$

$0 \le \rho \le 2\cos\varphi$ AND

$0 \le \rho \le 1/\sin\varphi$

$$0 \le \rho \le \min\left(2\cos\varphi, \frac{1}{\sin\varphi}\right.$$

$$\int_{\varphi=0}^{\varphi=\pi} \left| \int_{\theta=0}^{\theta=2\pi} \left| \int_{\rho=0}^{2\cos\varphi} f\cdot(\ )\, d\rho \right| d\vartheta \right| d\varphi$$

ρ^5 $\qquad \rho^2 \sin\varphi$

$$\int_0^{2\cos\varphi} \rho^7 \sin\varphi\, d\rho$$

$$= \frac{\rho^8}{8} \sin\varphi \Big|_0^{2\cos\varphi}$$

$\dfrac{1}{\sin\varphi}$!

very big $\qquad \sqrt{2} \qquad 1/1$

$(\varphi = 0) \qquad \varphi = 45^\circ \qquad \varphi = 90^\circ$

near $\varphi = 0$

Is $2\cos\varphi \overset{?}{\le} \dfrac{1}{\sin\varphi}$

$$\underbrace{2\cos\varphi \sin\varphi}_{\sin(2\varphi)} \overset{?}{\le} 1$$

$\sin(2\varphi) \le 1$ yes

128

$$\bigcap \int_{\varphi} \frac{2^8}{8} (\cos^8\varphi)(\sin\varphi)\, d\varphi$$

$$\left(\frac{2^8}{8} \frac{(\cos^9\varphi)}{9} (-1) \right)'$$

$\cos^8\varphi \, (-\sin\varphi)$

$=$

Median 14/21