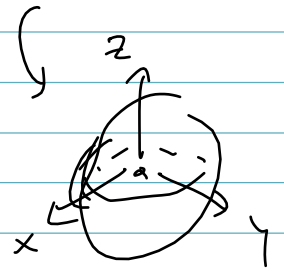
Physics: $\rho = \sqrt{x^2+y^2+z^2}$



$\rho = \sqrt{x^2+y^2+z^2}$

$z = \rho \cos\varphi$

$\langle 0,0,1\rangle$ $\varphi = 0$

$\langle 0,0,-1\rangle$ $\varphi = 180^\circ$ or π

φ = latitude

except ↑ North $\varphi = 0$

South ↓ $\varphi = 180^\circ = \pi$

$\theta = \vartheta$ = polar coordinates

Earth ϑ = longitude measurement

φ $0 \uparrow$ $\sin\varphi \geq 0$
$180 \downarrow$

Math 200, Nov 25

Cylindrical & Spherical Coords

Cylindrical Coords...

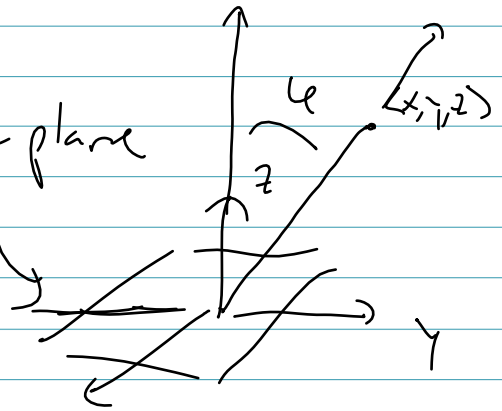
$z = z$ $x = r\cos\theta$ $y = r\sin\theta$

polar coords

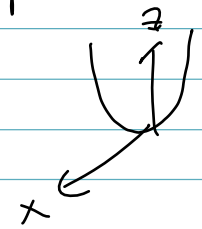
Periodic Table

2	1
8	4
8	4
18	9
18	9
32	16

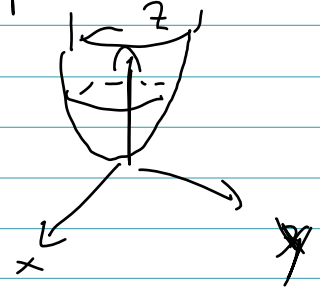
In xy-plane



parabola $z = 10x^2$

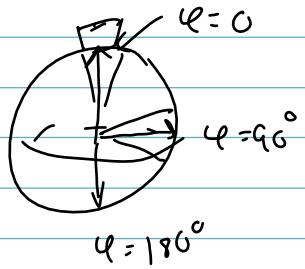


paraboloid $z = 10(x^2+y^2)$



$$dV = \rho^2 \sin\varphi \, d\rho \, d\vartheta \, d\varphi$$

polar coordinates



$$r = \rho \sin\varphi = \sqrt{x^2+y^2} \geq 0$$

$$\sin\varphi \geq 0$$

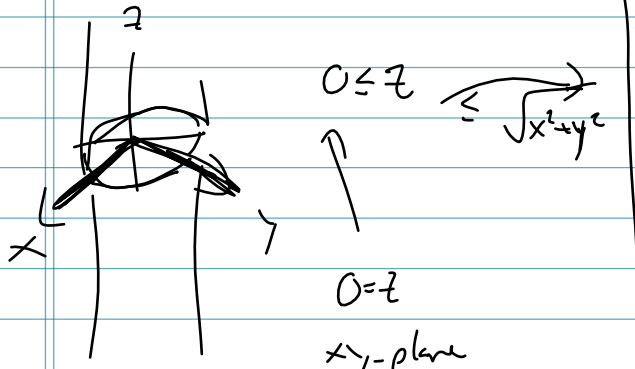
=

Earlier

$z = f(x)$ (rotate) $z = f(x^2+y^2)$

$x = r\cos\theta$

$y = r\sin\theta$



Problem 8 2013 WT2

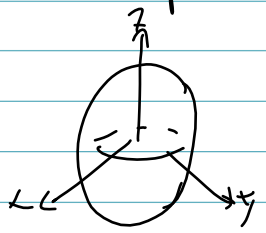
Solid:

$$\{(x,y,z) \mid 0 \le z \le \sqrt{x^2+y^2},\ x^2+y^2 \le 1\}$$

$I = \iiint f(x,y,z)\, dV$

dV = in cylindrical $dz\,(r\,dr\,d\theta)$

in spherical $\rho^2 \sin\varphi\, d\rho\, d\theta\, d\varphi$



$\rho = \sqrt{x^2+y^2+z^2}$

$z = \rho\cos\varphi$

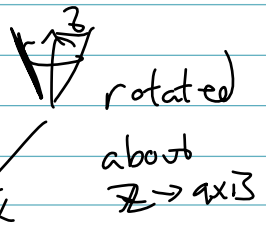
$(r = \rho\sin\varphi)$

Method 1:

Ch 12.6

$z = \sqrt{x^2+y^2}$

(should be $z = \sqrt{x^2}$)



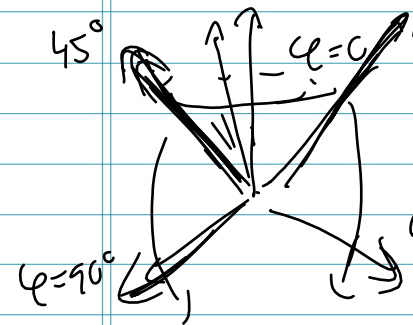
$1 \le 0$ ☹

$0 \le \rho\cos\varphi \le \rho\sin\varphi$

$z = \rho\cos\varphi$

$\sqrt{x^2+y^2} = r = \rho\sin\varphi$

$0 \le z \le \sqrt{x^2+y^2}$



☺

$\rho\cos\varphi \le \rho\sin\varphi$

$\cos\varphi = \sin\varphi$

$x^2+y^2 \le 1$

$r^2 \le 1$

$r \le 1$, $\rho\sin\varphi \le 1$