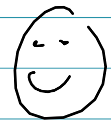
Lazy evaluation = most direct approach



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x appears in $(*)$, $(**)$

y " " "

z " in only $(**)$

try $dV = dz \begin{Bmatrix} dx\,dy \\ dy\,dx \\ r\,dr\,d\theta \end{Bmatrix}$

first later

$$\text{Volume} = \iiint_R 1 \cdot \underbrace{dV}_{dz(\quad)}$$

Math 200, Nov 23

→ Cylindrical coords

→ Spherical "

→ Review: past exam problems

Won't cover 15.10 Change of variables

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Last time: 2013 WT 1

Prob 7: Let $a > 0$, solid

inside $x^2 + y^2 - ax = 0 \quad (*)$

inside $x^2 + y^2 + z^2 = a^2 \quad (**)$

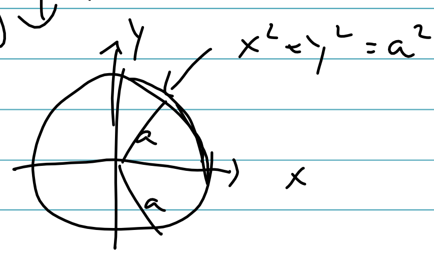
Find volume.

Hint: $\int \sin^3(\theta)\,d\theta = \ldots$

inside $x^2 + y^2 - ax = 0$

inside $[\quad \leq z \leq \quad]$

$(x^2 + y^2 \leq a^2)$



$$\boxed{(x^2 + y^2) - ax = 0}$$

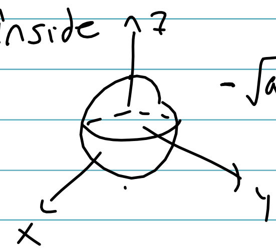
$$\left(x - \frac{a}{2}\right)^2 + y^2$$

$$= x^2 - ax + \frac{a^2}{4} + y^2 = \frac{a^2}{4}$$

First how z varies as a function of x, y

$$z = \pm\sqrt{a^2 - x^2 - y^2}$$

inside

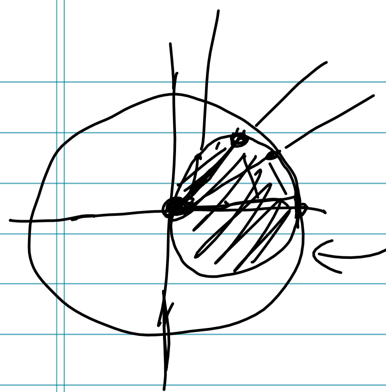


$$-\sqrt{a^2 - x^2 - y^2} \leq z \leq \sqrt{a^2 - x^2 - y^2}$$

$$\int_{z=\text{lower}}^{z=\text{upper}} 1\,dz = \text{upper} - \text{lower}$$

here: $\sqrt{a^2 - x^2 - y^2} - \left(-\sqrt{a^2 - x^2 - y^2}\right)$

$$= 2\sqrt{a^2 - x^2 - y^2}$$



in polar coords

$0 \leq r \leq \text{func}(\theta) = a\cos\theta$

where $x^2 + y^2 = ax$

$r^2 = ar\cos\theta$

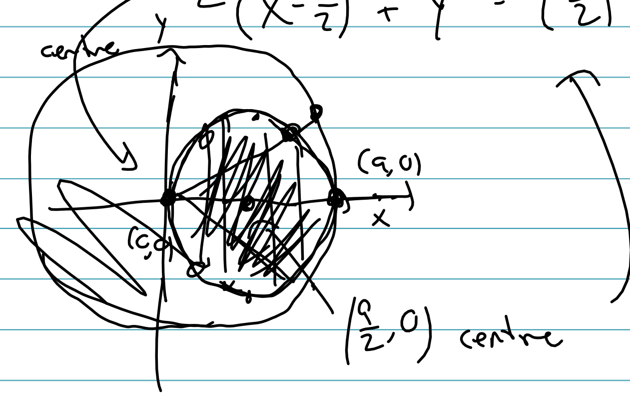
$r = a\cos\theta$

$$\iiint_R 1 \cdot dV$$

$$\iint_R 2\sqrt{a^2 - x^2 - y^2} \begin{Bmatrix} dxdy \\ \vdots \end{Bmatrix}$$

R inside $x^2 + y^2 = a^2$

$(x - \frac{a}{2})^2 + y^2 = (\frac{a}{2})^2$



$$\int \sin(t^2)\,dt =$$

$$\int \sin(u)\,du = -\cos(u)$$

$$\int 2t \sin(t^2) = -\cos(t^2)$$

$$\int e^{t^2}\,dt$$

$$\int 2t\, e^{t^2}\,dt = e^{t^2}$$

$$\int f(t)\,dt = F(t)$$

$$\int f(t^2)\,dt$$

Volume =

$$\int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{r=0}^{r=a\cos\theta} 2\sqrt{a^2-x^2-y^2}\; r\,dr\,d\theta$$

$$= \int_{x=0}^{x=a} \int_{y=\text{bottom}(x)}^{y=\text{top}(x)} 2\sqrt{a^2-x^2-y^2}\; d\cancel{y}\,dx$$

$$x^2+y^2 = ax \rightarrow y^2 = ax - x^2$$

$$\underbrace{-\sqrt{ax-x^2}}_{\text{bottom}(x)} \le y \le \underbrace{+\sqrt{ax-x^2}}_{\text{top}(x)}$$

$$= -\frac{(a-u)^{3/2}}{3/2}$$

$$\int a^p\,da = \frac{a^{p+1}}{p+1}$$

$$\int 2r\sqrt{a^2-r^2}\,dr$$

$$= -\frac{(a^2-r^2)^{3/2}}{3/2}$$

$$-\frac{(a^2-r^2)^{3/2}}{3/2}\Bigg|_{r=0}^{r=a\cos\theta}$$

$$= -\frac{(a^2-(a\cos\theta)^2)^{3/2}}{3/2} - -\frac{(a^2-0)^{3/2}}{3/2}$$

$$\int 2t\, f(t^2)\,dt = F(t^2)$$

$$\int 3t\, f(t^2)\,dt$$

$$= \int \frac{3}{2}\left(2t\, f(t^2)\,dt\right) = \frac{3}{2} F(t^2)$$

$$= \int_{\theta=-\pi/2}^{\theta=\pi/2} \left(\int_{r=0}^{r=a\cos\theta} 2\sqrt{a^2-r^2}\; r\,dr \right) d\theta$$

$$\int \sqrt{a-u}\,du = \int (a-u)^{1/2}\,du$$

$$= \int_{\theta=0}^{\theta=\pi/2} \sin^3\theta \, d\theta$$

$$+ \int_{\theta=-\pi/2}^{\theta=\text{[illegible]}} (-\sin^3\theta) \, d\theta$$

OR

$$= 2\int_{\theta=0}^{\theta=\pi/2} \sin^3\theta \, d\theta$$

Accident: $dV = dz \left\{ \begin{matrix} dx\,dy \\ ? \end{matrix} \right\}$

$$= dz\,(r\,dr\,d\theta)$$

$$= -\frac{\left(a^2(1-\cos^2\theta)\right)^{3/2}}{3/2} + \frac{a^3}{3/2}$$

$$= -\frac{a^3(\sin^2\theta)^{3/2}}{3/2} + \frac{a^3}{3/2}$$

$$= \frac{-a^3|\sin^3\theta|}{3/2} + \frac{a^3}{3/2}$$

$$\int \sin^3\theta \, d\theta = \frac{1}{12}\cos(3\theta) - \frac{3}{4}\cos(\theta) + C$$

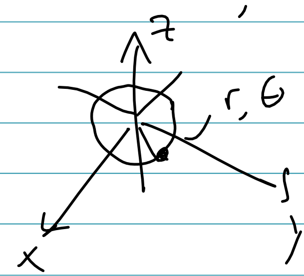
$$\int_{\theta=-\pi/2}^{\theta=\pi/2} |\sin^3\theta| \, d\theta$$

Polar Coords

$$\begin{matrix} dx\,dy \\ dy\,dx \end{matrix} = (r\,dr\,d\theta)$$

3-d:

Fix z, then $\overbrace{r\,dr\,d\theta}^{x,y}$



cylindrical coords