Math 200, Nov 20

2013 WT2

Prob 6 (b) $\iiint$

$dz\,dy\,dx \to dx\,dy\,dz$

6 (a)

$$\int_0^2 \int_0^x f(x,y)\,dy\,dx + \int_2^6 \int_0^{\sqrt{6-x}} f\,dy\,dx$$

$dy\,dx \to dx\,dy$

$$\int_{x=0}^{x=2} \left(\int_{y=0}^{x} \text{y varying}\ f_i x\,x \right) \text{vary } x$$

Recall $\iiint$ also $\iint$

lazy evaluation ☺

=

$dx\,dy \to dy\,dx$

(often faster to draw picture)

$dz\,dy\,dx \to dx\,dy\,dz$

formally look at the inequalities

$0 \le y \le 2$

$y \le x \le \left(\begin{array}{c} y=\sqrt{6-x} \\ y^2 = 6-x \end{array} \right)$

$x = 6-y^2$

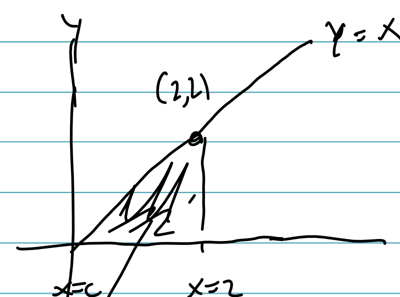
$y \le x \le 6-y^2$

Formally:

Region

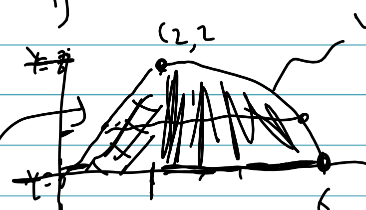
$0 \le x \le 2$	$2 \le x \le 6$
$0 \le y \le x$	$0 \le y \le \sqrt{6-x}$

FORGET



$0 \le x \le 2$

$0 \le y \le x$



$2 \le x \le 6$

$0 \le y \le \sqrt{6-x}$

$0 \le y \le 0$ $\quad x=6$

$0 \le y \le \sqrt{4}$ $\quad x=2$

Is this the same?

We think: $0 \le y \le 2$

anything

$$y \le x \le 6-y^2$$

what if

$$y > 6-y^2$$

Could be that

want to know that for any

$0 \le y \le 2 \to y \le 6-y^2$

I

$0 \le x \le 2$

$0 \le y \le x$

II

$2 \le x \le 6$

$0 \le y \le \sqrt{6-x}$

$$f_1(y) \le x \le f_2(y)$$

$$c \le y \le d$$

$$y \le x \le 6-y^2$$

$x \in [0,2]$

$y \le \sqrt{6-x}$

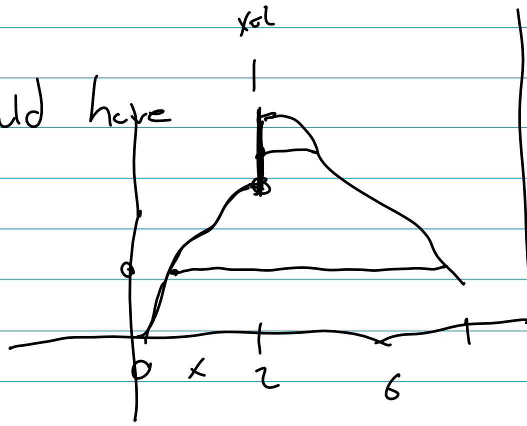
$y^2 \le 6-x$

$x \le 6-y^2$

looks like

$c \le y \le d \to ?$ $0 \le y \le \left\{ \begin{matrix} 2 \\ 2 \end{matrix} \right\}$

Could have



Draw picture...

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2013 WT I

Prob: 7. Fix $a > 0$

Check:

$$y \le 6-y^2$$

where we want?

$$y^2 + y \le 6$$

$$\left(y+\tfrac{1}{2}\right)^2 = y^2 + y + \tfrac{1}{4} \overset{?}{\le} 6 + \tfrac{1}{4}$$

$$\left(y+\tfrac{1}{2}\right)^2 \le \frac{25}{4}$$

$$\left|y+\tfrac{1}{2}\right| \le \frac{5}{2}$$

Is true for $0 \le y \le 2$

Yes...

Cylinder $x^2+y^2 \le ax$

$$(x^2-ax)+(y^2) \le 0$$

$$\left(x-\tfrac{1}{2}a\right)^2+y^2$$

$$=\left(x^2-ax+\frac{a^2}{4}\right)+y^2 \le \frac{a^2}{4}$$

$$\boxed{\left(x-\frac{a}{2}\right)^2+y^2 \le \left(\frac{a}{2}\right)^2}$$

$\iiint dV$ $\quad dx\,dy\,dz$ $\quad dz\,dy\,dx$ $\quad \vdots$ $\quad \Big|\ dz \begin{cases} dx\,dy \\ r\,dr\,d\theta \end{cases}$ $\quad x,y$ $\quad r,\theta$

Inside sphere

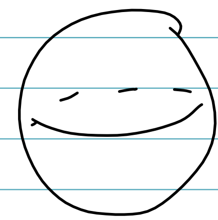
$$x^2+y^2+z^2 \le a^2$$

Inside cylinder

$$x^2+y^2 = ax$$

$$x^2+y^2 \le ax$$

= Compute Volume



$$\boxed{x^2+y^2+z^2 \le a^2}$$

$$\int_{z=-\sqrt{a^2-x^2-y^2}}^{z=\sqrt{a^2-x^2-y^2}} dz$$

later

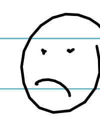
$$\iint_{x,y} \Big(\quad \Big)\, dx\,dy \quad dy\,dx$$

circle interior

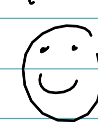
$x^2+y^2 \le a^2$

Think lazy

$x,y \rightsquigarrow$ both inequalities



$z \rightsquigarrow$ only in one



$$x^2+y^2+z^2 \le a^2$$

$$z \quad -\sqrt{a^2-x^2-y^2} \ \text{to} \ +\sqrt{a^2-x^2-y^2}$$

$$\iint_A \left(\int dz \right) r\,dr\,d\theta$$

better

$dx\,dy$

"cylindrical coordinates"

$$\int_{z=}^{z=} dz = z \Big|_{z=-\sqrt{a^2-x^2-y^2}}^{z=\sqrt{a^2-x^2-y^2}}$$

$$= 2\sqrt{a^2-x^2-y^2}$$

Integral 2-d: $\displaystyle\iint_{x^2+y^2\le a^2} 2\sqrt{a^2-x^2-y^2}\ \overbrace{\begin{Bmatrix} dx\,dy \\ dy\,dx \end{Bmatrix}}^{dA}$

$r\,dr\,d\theta$ $\quad \begin{cases} r = 0, a \\ \theta = 0, 2\pi \end{cases}$