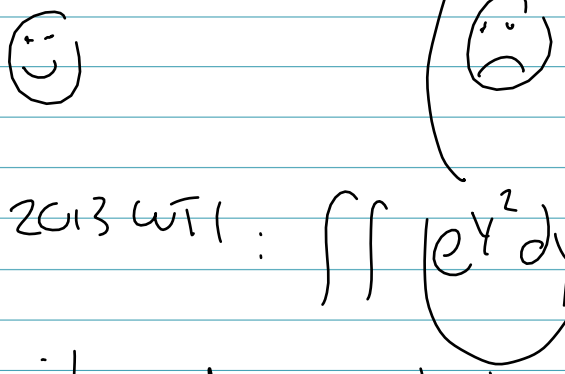
Themes:

lazy vs. un-lazy vs impossible evaluation



2013 WT1: $\iint e^{y^2}\,dy\,dx$

idea $dy\,dx \to dx\,dy$

$=$

$0 \le x \le 1$, $0 \le y \le 2$ $\quad \int_0^2 \left(\int_0^1 f\,dx \right) dy$

Which way?

(1) First hold y, z const
~~Then~~ vary x

(2) Hold x, z constant
any vary y

(3) Hold x, y const
vary z

$=$ (1) $\int (\text{const})\, e^{-x(\text{const}')}\,dx$ (lazy)

Math 200, Nov 18

$\iiint$ — cylindrical
— spherical

— Change of Variables

— Rest: Old exam problems

$\iiint dV$ almost nothing new...

One new idea: $dx\,dy\,dz$
to $dy\,dz\,dx$
to ⋮

Even work: $dx\,dy \leftrightarrow dy\,dx$

2012 WT1:

$\iiint y z^2 e^{-xyz}\,dV$

$dV = dx\,dy\,dz$ ←

R: Rectangular Box — $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$

$dV = dx\,dy\,dz$ or $dy\,dx\,dz$ or (6 ways)

9.6

$$\iiint \boxed{yz^2 e^{-xyz}}\, dV$$

$\int e^{-xC_2}(-C_2)\,dx = e^{-xC_2}$

$\frac{C_1}{-C_2}\int \dots = \frac{C_1}{-C_2} e^{-xC_2}$

$= \int C_1 e^{-xC_2} =$

$= \iint \left(\int_{\text{const}} yz^2 e^{-x\,yz}\, dx \right) dy\, dz$

$\left(\frac{yz^2}{-yz} e^{-xyz} \right|_{x=0}^{x=1}$

(2) Fix x, z

$\int y\,(\text{const})\, e^{-y(\text{other constants})}\, dy$

(3) $\int z^2(\text{other const}) e^{-z\,\text{Const}}\, dz$ (work hard)

$\int C_1 e^{-xC_2}\, dx$

$(e^{-xC_2})' = e^{-xC_2}(-C_2)$

$z\cdot 2 - (-1)e^{-2z}$ $(y=2)$

$-\left(0 - (-1)\,1\right)$ $(y=0)$

$= 2z + e^{-2z} - 1$

$\int_{z=0}^{z=3} 2z + e^{-2z} - 1\, dz$

$= \dots$

Normal: $\int e^{y^2} dy$ ☹

$\iint \left(-z e^{-xyz} \Big|_{x=0}^{x=1} \right) dy\, dz$

$\int \int \left(-z e^{-yz} - (-z) \right) dy\, dz$

$\int \int \left(z - z e^{-yz} \right) \begin{Bmatrix} dy\, dz \\ dz\, dy \end{Bmatrix}$

$\int_{y=0}^{y=2} (z - z e^{-yz})\, dy$

$\int \left(yz^2 e^{-xyz} dx \right)$

$\left(zy - \left(\frac{z}{-z}\right) e^{-yz} \right) \Big|_{y=0}^{y=2}$

$0 \le z \le y$

$0 \le y \le x$

$0 \le x \le 1$

$0 \le z \le y \le x \le 1$

$\int\int\left(\int \cdot dx\right) dy\, dz$

$\text{first}(y,z) \le x \le \text{first}(y,x)$

Fix y,z

x

Fix y,x

Nothing fixed — drew a picture

x varies — another way

2013 WT2

6 (a)

$dy\,dx \to dx\,dy$

$=$

(b) $\int_0^1\left(\int_0^x\left(\int_0^y f(x,y,z)\, dz\right) dy\right) dx$

$dz\,dy\,dx \to (((dx)\,dy)\,dz)$

fix one variable

other two vary

$0 \le z \le y \le 1$

$\int_z^1\left(\int f\, dx\right) dy$

$\int_{z=0}^{z=1}\left(\int\int dx\,dy\right) dz$

$0 \le z \le y \le x \le 1$

$\int_{z=0}^{z=1}\int_{y=z}^{y=1}\int_{x=y}^{x=1}$

Fix y,z

$0 \le \text{fixed} \le \text{something else} \le x \le 1$

fixed

$y \le x \le 1$

$\int_{z=0}^{\text{fixed}(z)}\left(\int_{x=y}^{x=1} f\, dx\right) dy$

$\text{fixed}(z)$

ignoring x: $0 \le z \le y \le$ ignoring ≤ 1