2010 WT2

3. Suppose $w = f(xz, yz)$.

Show $\boxed{x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} = z\frac{\partial w}{\partial z}}$

1 $w = f(u,v)$ and $u = xz$, $v = yz$ | u,v | x,y,z

$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x} f(u, v)$, $u(x,y,z) = xz$, $v = v(x,y,z) = yz$

Background

$= f_u u_x + f_v v_x$

$\left(f_1 \frac{\partial(xz)}{\partial x} + f_2 \frac{\partial(yz)}{\partial x} \right)$

Math 200, Nov 9:

- Review Ch 14 for midterm on
- Friday: 2010 WT1, 2010 WT2!
- Location of midterm: To Be Announced
- Office hours: Tu, Th afternoon
  Tu: 4:45 – 5:30pm
  Th: " – 5:45pm

Midterm: Ch 14:

- ∇f, Directional Derivative, Linear Approx
- Chain Rule: Concrete, General Type
- Max/min critical points
- Lagrange multipliers

Later parts of Ch 14

$\frac{\partial w}{\partial x} = f_u \cdot z$ STOP!

$\frac{\partial w}{\partial y} = \frac{\partial}{\partial y} f(u,v)$ $\begin{cases} u = xz \\ v = yz \end{cases}$

$= f_u \frac{\partial}{\partial y}(xz) + f_v \frac{\partial}{\partial y}(yz)$

$= f_u\, 0 + f_v\, z$ STOP!

$w_x = f_u z$, $w_y = f_v z$

$w_z = f_u u_z + f_v v_z$

$= f_u (xz)_z + f_v (yz)_z$

$= f_u x + f_v y$ STOP!

$\frac{\partial(xz)}{\partial x} = \left(\frac{\partial}{\partial x} x\right) z + x \left(\frac{\partial}{\partial x} z\right)$

z, y constants, x varying

$= z$

$f = f_u u_x + f_v v_x$ general

$= f_u z + f_v \cdot 0 = f_u z$

$f_u = f_1 = D_1 f$

Could be $f(u,v) = \begin{cases} u^{10} + 3v^2 \\ \sin(vu) + 3 \\ \vdots \end{cases}$

$f_u = ?$ $f_v = ?$

$\frac{\partial}{\partial p}$: p varying, T fixed

$$\underline{V = V(p,T)}$$

$$\Delta V = V_p \Delta p + V_T \Delta T$$

(1) We want $V_p = \frac{\partial V}{\partial p}$
and $V_T = \frac{\partial V}{\partial T}$ at $\boxed{(p,V,T) = (1,2,5)}$

Implicit differentiation

$$\frac{\partial}{\partial p}\left((pV^2+16)(V-1) = TV^2 \right)$$

$$\left(\frac{\partial}{\partial p}(pV^2+16)\right)(V-1) + (pV^2+16)\frac{\partial}{\partial p}(V-1) = \frac{\partial}{\partial p}(TV^2)$$

We have

$w_x = f_u z$, $w_y = f_v z$,

$w_z = f_u x + f_v y$

Hopefully:

$$x\, w_x + y\, w_y \overset{?}{=} z\, w_z$$

$$x \cdot f_u z + y \cdot f_v z = z \cdot (f_u x + f_v y)$$

(2) W2010T1

$$(pV^2+16)(V-1) = TV^2$$

Prob: What ΔV at $(1,2,5)$ for some ΔT, Δp

At $(p,V,T) = (1,2,5)$

$$(4 + 1 \cdot 2 \cdot 2\, V_p) \cdot 1 + (1 \cdot 4 + 16) V_p = 5 \cdot 2 \cdot 2 \cdot V_p$$

$V_p =$

$V_T =$

$$\Delta V = V_p \Delta p + V_T \Delta T$$

$\vdots$

$$\frac{\partial}{\partial p}(pV^2+16) = (pV^2)_p$$

$$= (p)_p V^2 + p(V^2)_p$$

$$= \boxed{1 \cdot V^2 + p \cdot 2V \cdot V_p}$$

$$\frac{\partial}{\partial p}(V-1) = V_p$$

$$\left[(pV^2+16)(V-1)\right]_p = \left[TV^2\right]_p$$

$$(V^2 + p\,2V \cdot V_p)(V-1) + (pV^2+16)V_p = T\,2V \cdot V_p$$