

Model Crescent:

Unit disc $(x^2 + y^2 \leq 1)$
 remove interior of circle
 $x^2 + y^2 = x$, Density = 1

(1) Find mass

(2) Find x-coord of
 centre of mass

Hint $\int_{-\pi/2}^{\pi/2} \cos^4(\theta) d\theta = \frac{3\pi}{8}$

(Polar coords $dA = r dr d\theta$)

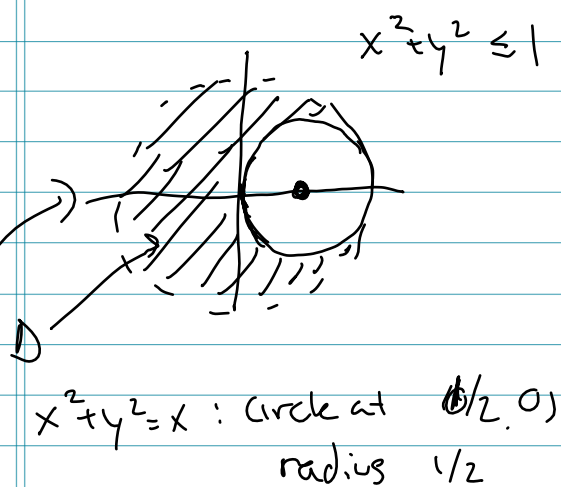
Math 200 Nov 6

- Midterm in one week
- Wednesday holiday
- Office hours next week: TuTh afternoon
- Practice to be posted today
- Monday! some review Ch. 14

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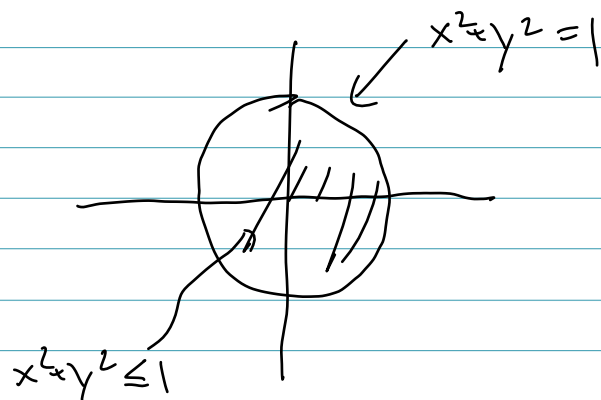
Centre of Mass in 2-d

$$\begin{matrix} \text{mass} \\ \bar{x} \\ \bar{y} \end{matrix} \left\{ \begin{array}{l} \iint \rho dA \\ \iint x \rho dA \\ \iint y \rho dA \end{array} \right.$$



mass = area = $\iint_D dA$
 (density 1)

area = $\left(\begin{array}{l} \text{area} \\ \text{of} \\ \text{big} \\ \text{circle} \end{array} \right) - \left(\begin{array}{l} \text{area} \\ \text{of} \\ \text{small} \\ \text{circle} \end{array} \right)$



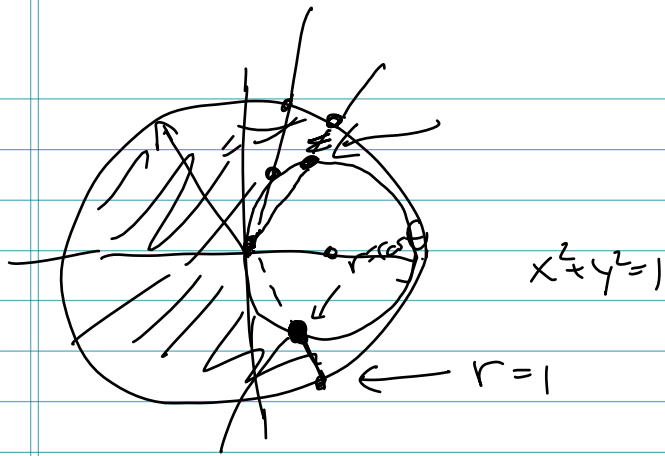
remove $(x^2 + y^2) = x \dots ?$

Q1: $(x^2 - x) + y^2 = 0$

Complete

$$\left(x - \frac{1}{2}\right)^2 + (y - 0)^2 = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

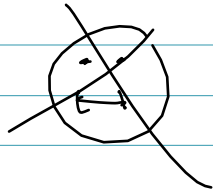
$$x^2 - x + \frac{1}{4} + y^2$$



big circle area (radius 1)
 $= \pi \cdot 1^2 = \pi$

small circle (radius 1/2) area
 $= \pi \cdot (1/2)^2 = \pi/4$

$$\iint_D dA = \pi - \frac{\pi}{4} = \frac{3}{4}\pi$$

= Polar coordinates  maybe ??

for $-90^\circ \leq \theta \leq 90^\circ$
 given θ :


$r \leq 1$

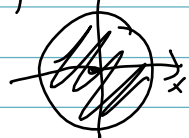
$x^2 + y^2 = x$

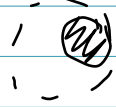
$x = r \cos \theta$
 $y = r \sin \theta$

$r^2 = (r \cos \theta)^2 + (r \sin \theta)^2 = r \cos \theta$

Remark:

$$\iint x \, dA$$


$$= \iint x \, dA$$


$$- \iint x \, dA$$


$r^2 = r \cos \theta$ given θ

$r = \cos \theta$ $-90^\circ \leq \theta \leq 90^\circ$

$0 \leq \theta \leq 90^\circ$, $\cos \theta \leq r \leq 1$

$90^\circ \leq \theta \leq 270^\circ$ $0 \leq r \leq 1$

$270^\circ \leq \theta \leq 360^\circ$ $\cos \theta \leq r \leq 1$

$$\bar{x} = \frac{1}{\text{mass}} \iint_D x \, dA \quad (\text{density } 1)$$

$$= \frac{1}{\frac{3}{4}\pi} \iint_D x \, dA$$

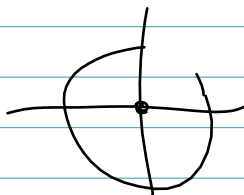
Fact

$$\left(\iint_{\text{circle}} x \, dA = \bar{x} = x_0 \right)$$

circle
centre
(x_0, y_0)

area of circle

$$\iint_{\text{circle}} x \, dA = 0$$



$$\left(\iint_{\text{circle}} x \, dA = (x\text{-coord centre}) \cdot (\text{area of circle}) \right)$$

$$\iint x \, dA$$



$$\left(\iint_{\text{Symm about } x} x \, dA \right) = (x \text{ where it is symm}) \cdot (\text{area})$$