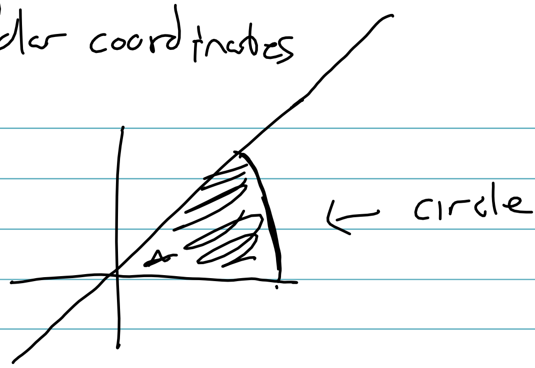
Polar coordinates



Why? - Certain regions work well

- Certain integrands

$\sin(x^2+y^2)\,dx\,dy$

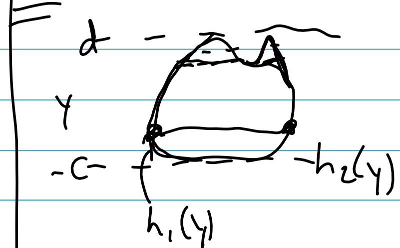
can't easily be integrated...

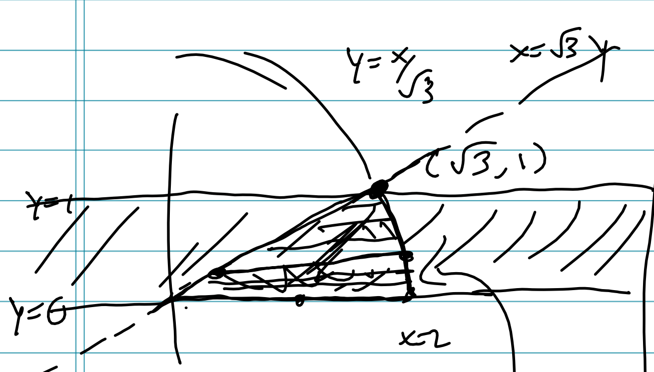
Math 200, Nov 4

Midterm 2: Ch 14 alone (14.2 omitted), no Ch 15

Ch 15.5 - applications of $\iint$

Only do mass, centre of mass





$y=0$: $\sqrt{3}\cdot 0 \le x \le \sqrt{4-0^2}$

$0 \le x \le 2$

$x^2+y^2=4$ circle

$y=1$: $\sqrt{3} \le x \le \sqrt{4-1}$

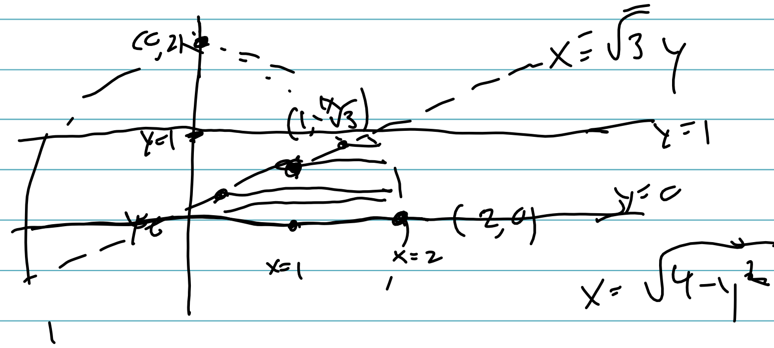
$\sqrt{3} \le x \le \sqrt{3}$

$x=\sqrt{4-y^2}$

2013 WT1 Prob 5:

$$\int_{y=0}^{y=1}\left(\int_{x=\sqrt{3}\,y}^{x=\sqrt{4-y^2}} \ln(1+x^2+y^2)\,dx\right)dy$$

(a) Sketch the domain



$x^2=4-y^2$

$x^2+y^2=4$

bottom line: $\theta = 0$

$y = \frac{1}{\sqrt{3}}x \qquad \theta = 30^\circ = \frac{180^\circ}{6} = \frac{\pi}{6}$

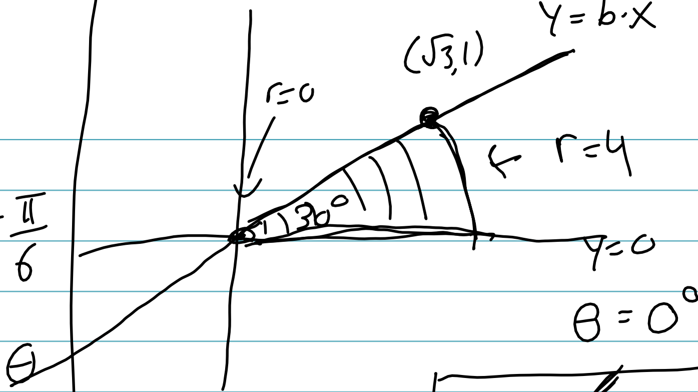
$x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2$

$x^2+y^2 = 4, \quad r = 2$

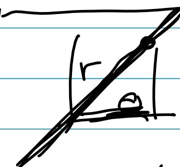
$$\int_{y=0}^{y=1}\int_{x=\sqrt{3}y}^{x=\sqrt{4-y^2}} dx\,dy$$

$$= \int_{\theta=0}^{\theta=\pi/6}\int_{r=0}^{r=2} r\,dr\,d\theta$$



$y = \sqrt{3}\,x$

$\tan\theta$



$x = (\cos\theta)\,r$

$y = (\sin\theta)\,r$

$y = bx$: θ s.t.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x} = \frac{bx}{x} = b$

$$\int_0^1\int_{\sqrt{3}y}^{\sqrt{4-y^2}} \ln(1+x^2+y^2)\,dx\,dy$$

$$= \int_{\theta=0}^{\theta=\pi/6}\left(\int_0^4 \ln(1+r^2)\,r\,dr\right)d\theta$$

$$\int \ln(x)\,dx = \text{something}(x)$$

$$\int f(x)\,dx = F(x)$$

$$F'(x) = f(x)$$

$$\frac{d}{dr} F(r^2) = 2r\,F'(r^2)$$

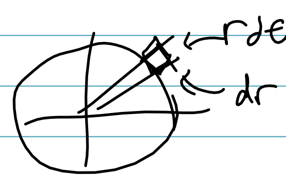
Moral...

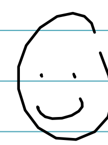
$y = \text{const}\cdot x \rightarrow \theta = \text{other constant}$

$x^2+y^2 = \text{const} \rightarrow r = \sqrt{\text{const}}$

$\ln(1+x^2+y^2)$ $dx\,dy$ / $dy\,dx$

$\ln(1+r^2)\,r\,dr\,d\theta$





$$\int (1+\ln x)\,dx = x\ln x$$

$$\int (\ln x)\,dx = x\ln x - x + C$$

$$\int 2r\,f(r^2)\,dr$$

substitution:

$u = r^2$

$du = 2r\,dr$

$$\frac{d}{dr}F(r^2) = 2r\,F'(r^2)$$