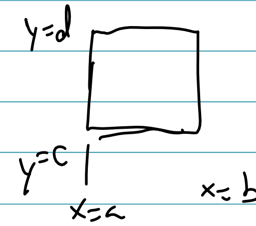
Math 200, Nov 2

15.1 Double integrals, the theory...

15.2 $\iint$ over rectangles



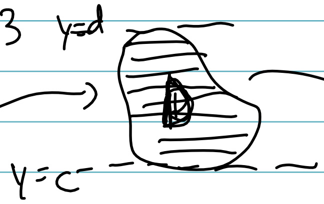
$$\underbrace{\iint f\, dx\, dy}_{} = \iint f\, dy\, dx$$

$$\int_{y=c}^{y=d}\left(\int_{x=a}^{x=b} f\, dx\right) dy$$

Ch 15 $dx\,dy \approx dy\,dx$

Ch 14 $f_{xy} = f_{yx}$

15.3



$$\iint_D f\, dA = \iint$$

Fix y, $c \le y \le d$

$$g_1(y) \le x \le g_2(y)$$

$$\int_{y=c}^{y=d}\left(\int_{x=g_1(y)}^{x=g_2(y)} f(x,y)\right) dy$$

Prob 5(a) Final 2013 WT2

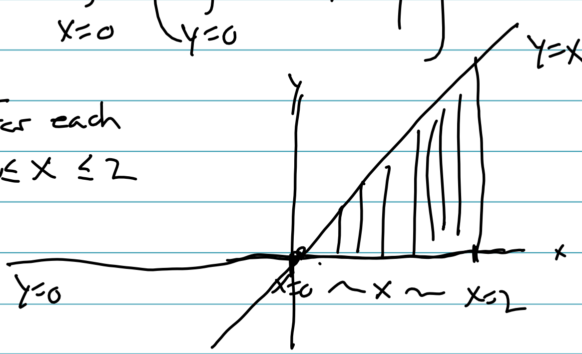
change $dy\,dx \to dx\,dy$

$$I_1 = \int_0^2\left(\int_0^x f(x,y)\,dy\right)dx$$

$$= \int_{x=0}^{2}\left(\int_{y=0}^{y=x} f(x,y)\,dy\right)dx$$

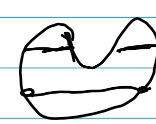
fix x, $\int$ in y, then $\int$ in x

For each $0 \le x \le 2$

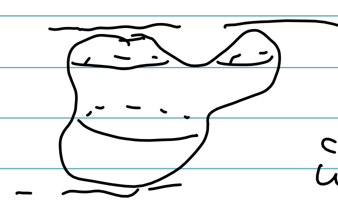


Tricky!

fix y



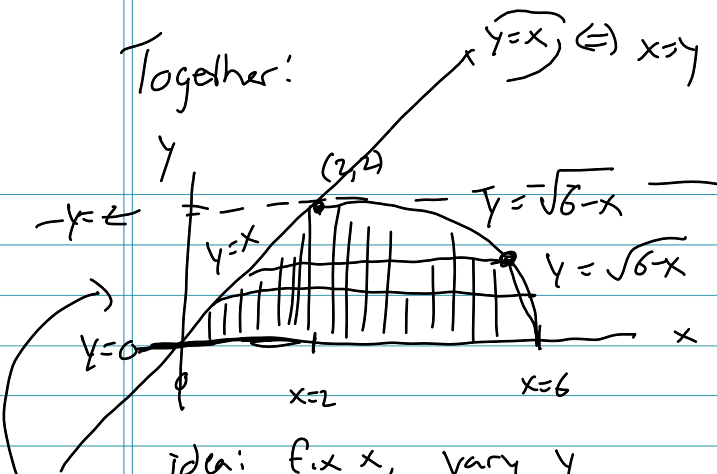
Tricky



3-d could be worse

$$I = \overbrace{\int_0^2\int_0^x f(x,y)\,dy\,dx}^{I_1} + \overbrace{\int_2^6\int_0^{\sqrt{6-x}} f(x,y)\,dy\,dx}^{I_2}$$

Together:



idea: fix x, vary y

fix y, vary x

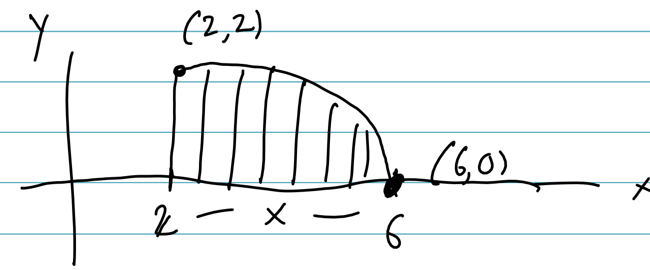
$0 \le y \le 2$

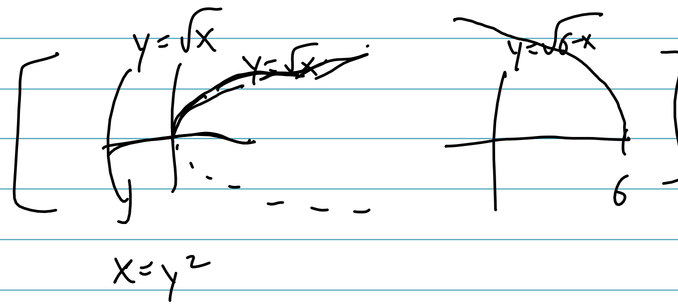
$y \le x \le$ (fund y)   ($y=\sqrt{6-x}$)

$y=\sqrt{6-x}$, $y^2 = 6-x$ or

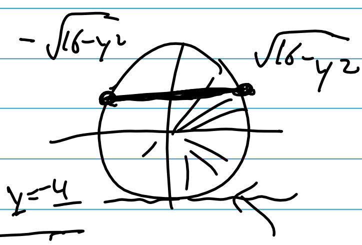
$x = 6-y^2$

$$I_2 = \int_{x=2}^{x=6}\left(\int_{y=0}^{y=\sqrt{6-x}} f(x,y)\,dy\right)dx$$



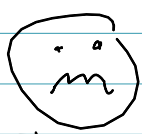


(1) fix y
vary x:



$x = \pm\sqrt{16-y^2}$ $x^2+y^2=16$

$$\boxed{1}\quad \int_{y=-4}^{y=4}\left(\int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} f(x,y)\,dx\right)dy$$



$x = \pm\sqrt{16-y^2}$

$$\boxed{2}\quad \int_{x=-4}^{x=4}\left(\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} f(x,y)\,dy\right)dx$$

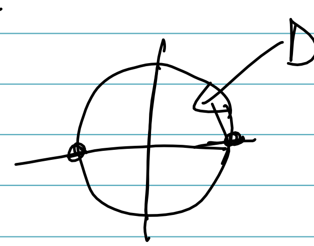
Hidden: $y=x$, $x=y$

Not so hidden

$y=\sqrt{6-x}$, $x = 6-y^2$

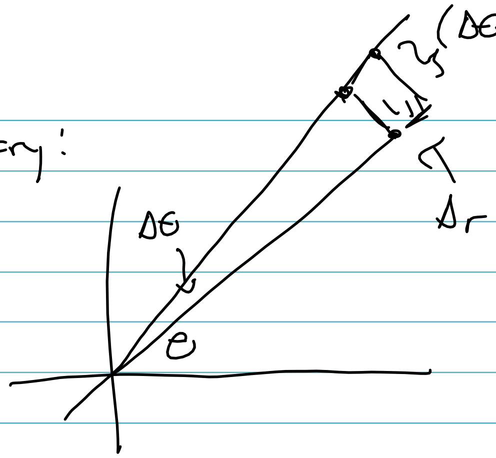
$$\int_{y=0}^{y=2}\left(\int_{y}^{6-y^2} f(x,y)\,dx\right)dy$$

=



$x^2+y^2 \le 16$

Theory:



Plug & Chug:

$$dA = dx\,dy \to r\,dr\,d\theta$$

15.10: more general way

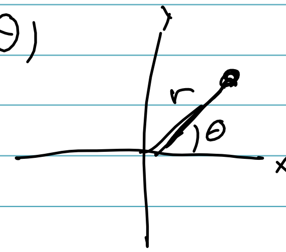
(x, y) to (r, θ)

to (u, v)

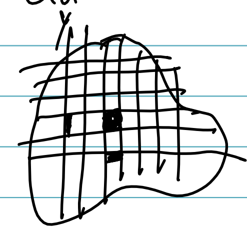
Often, with circles, cylinders, ...

Work with polar coordinates

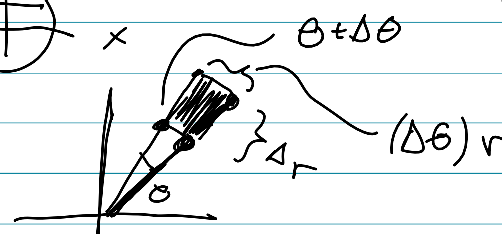
$$(x, y) \to (r, \theta)$$



old!



each $\Delta A = \Delta x \cdot \Delta y$



$$\Delta A \approx \underline{r\,\Delta r\,\Delta\theta}$$

$$\iint e^{-x^2-y^2}\,dA$$

(Interior of circle)

$$= \int_{\theta=0}^{\theta=2\pi}\left(\int_{r=0}^{4} e^{-x^2-y^2}\,r\,dr\right)d\theta$$

$$= \int_{r=0}^{r=4}\left(\int_{\theta=0}^{\theta=2\pi} e^{-x^2-y^2}\,r\,d\theta\right)dr$$

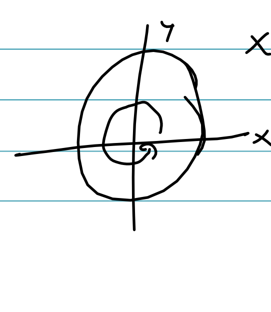
$0 \le r \le 4$ $\quad$ $0 \le \theta \le 2\pi$

Problem: Evaluate

$$\iint e^{-x^2-y^2}\,dx\,dy$$

in the region $D = \{x^2+y^2 \le 16\}$

[Hint: You cannot directly integrate $e^{-x^2}dx$, but you can re^{-r^2}, so try polar coordinates.]



$x^2+y^2 \le 16$ $\quad$ (360°)

Polar $0 \le \theta \le 2\pi$

$0 \le r \le 4$