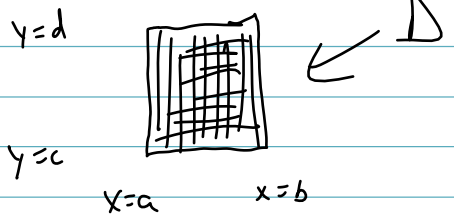
Ch 15: Double integrals:

$f = f(x,y)$



$$\int_D f(x,y)\,dx\,dy$$

should be: (average of f over D)(area)

or

$$\frac{\int_D f(x,y)\,dx\,dy}{\text{area } D} = \text{average of } f \text{ over } D$$

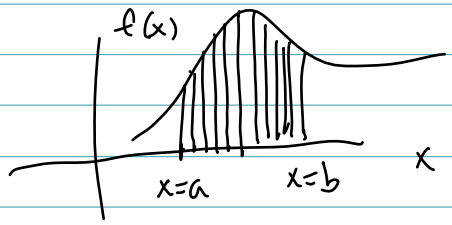
Ch 15

15.3 :

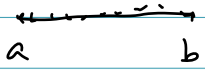
15.1 : Theory

15.2: Rectangular Double Integrals

Back 1-dim:

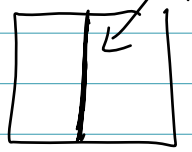


$$\int_{x=a}^{x=b} f(x)\,dx$$



$\approx$ Area of region from 0 to $f(x)$ via small rectangles

$$\int \left(\int_0^3 f\,dy \right) \text{in } x$$



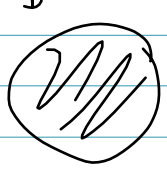
fix $0 \le x \le 2$

$$\int_{x=0}^{x=2} \left(\int_{y=0}^{y=3} f(x,y)\,dy \right) dx$$

15.1 & 15.2 warmup...

$=$

15.3



$$\boxed{\int_D f(x,y)\,dx\,dy}$$

$0 \le y \le 3$

D: $0 \le x \le 2$

$f(x,y) = x^2 y$

$=$

$$\int \left(\int \text{ dx each } y \right) \text{in } y$$

$$\int_c^3 \left(\int_0^2 f(x,y)\,dx \right) dy$$

15.3

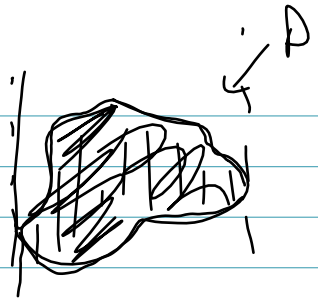
$\int x^2 y \, dA$ $\quad 0 \le y \le 3$, $\quad 0 \le x \le 2$

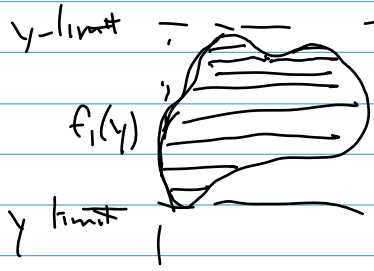
$\int x^2 y \, dx \, dy$

$\int_{y=0}^{3} \left(\int_{x=0}^{x=2} x^2 y \, dx \right) dy$

more gen

$\int_{y=0}^{3} \left(\text{but} \int_{x=\text{func}(y)}^{x=\text{other func}(y)} dx \right) dy$





$\int_{y=\text{lower}}^{y=\text{upper}} \left(\int_{x=f_1(y)}^{x=f_2(y)} \text{integrand} \, dx \right) dy$

$\int_{x=0}^{x=2} x^2 (y) \, dx$ $\quad$ x varying, y constant

$\int x^2 \, dx = \frac{x^3}{3} + C$ $\quad \left(\int x^n = \frac{x^{n+1}}{n+1} \right)$

$\int x^2 y \, dx$ (y fixed)

$= \frac{x^3 y}{3} + C$

$\int_{x=0}^{x=2} x^2 y \, dx = \frac{x^3 y}{3} \Big|_{x=0}^{x=2}$

$= \frac{2^3 y}{3} - \frac{0^3 y}{3} = \frac{8y}{3}$

func of y

$\int_{y=0}^{y=3} \left(\frac{8y}{3} \right) dy$

$= \frac{8}{3} \frac{y^2}{2} \Big|_{y=0}^{y=3}$

$= \frac{8}{3} \left(\frac{9}{2} - \frac{0^2}{2} \right) = \frac{8 \cdot 9}{3 \cdot 2}$

$= \frac{72}{6} = 12 \ldots$