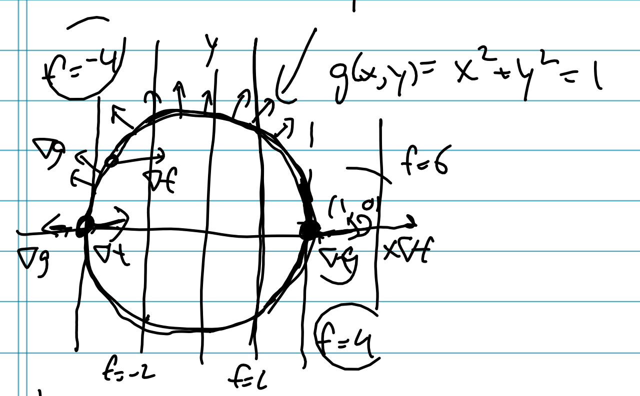
e.g.

min/max $f(x,y) = 4x$)

subject to $x^2+y^2=1$



No fine print...

typically have no boundary to check

... bounded region

Key observation:

14 End: Lagrange Multipliers

Test)

- Notes from class will probably go on the web...

Lagrange Multipliers:

Back 14.7: Min/max $f = f(x,y)$ or $f = f(x,y,z)$ set $\nabla f = 0$.

[Fine print: We have to check boundary...]

We ask min/max f subject to constraint: $g(x,y) =$ constant

$g(x,y,z) =$ constant

such as

$x^2+y^2+z^2 =$ constant

Upshot: instead of

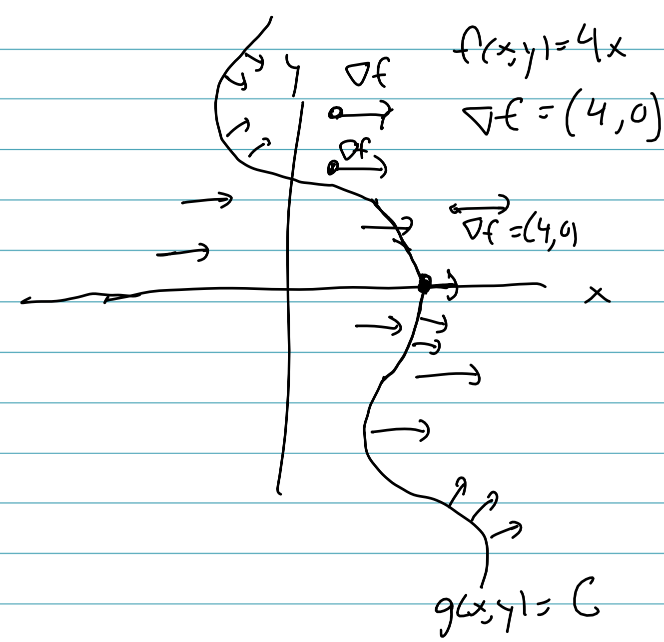
14.7 $\nabla f = 0$ for local min/max

14.8 min/max f s.t. $g(x,y) = C$

solve $\underbrace{\nabla f = k \nabla g}_{\nabla f,\ \nabla g \text{ parallel}}$, $g(x,y)=C$

Remark (not promise)
2013WT2, 2013WT1, 2012WT1, ~
Wikipedia, ~

$g(x,y,z) = x^2+y^2+z^2 = C$
(also $x \geq 0$)



∇g is $\perp$ to level curves of g

If $f(x,y)$ has local max/min at $g(x,y)=C$, $\nabla f, \nabla g$ parallel there.

Min/max

$$f(x,y,z) = 6x + y^2 + xz$$

subject to

$$g(x,y,z) = x^2+y^2+z^2$$

limit ourselves to $g(x,y,z) = 36$

=

$\nabla f, \nabla g$ parallel, i.e. $\nabla f = k \nabla g$

$g(x,y,z) = 16$

$$\nabla f = (f_x, f_y, f_z) = (6+z, 2y, x)$$

$$\nabla g = (g_x, g_y, g_z) = (2x, 2y, 2z)$$

So

$$\nabla f = k \nabla g$$

$$\begin{cases} f_x = k\, g_x \\ f_y = k\, g_y \\ f_z = k\, g_z \end{cases} \quad g(x,y,z) = 36$$

$$\begin{aligned} 6+z &= k\, 2x \\ 2y &= k\, 2y \\ x &= k\, 2z \end{aligned} \quad \Bigg| \quad x^2+y^2+z^2 = 36$$

$2y = k\,2y$

means $\begin{cases} k=1 \\ \text{or} \\ y=0 \end{cases}$

Case 1: $k=1$ (looks easy)

Case 2: $y=0$ (" ")

=

$k=1$: $6+z = 2x$ | $x^2+y^2+z^2 = 36$

$(2y = 2y)$ | $\cancel{x^2+z^2=16}$

$x = 2z$ | $y=0$

=

$x = 2z$, $2x = 4z$

$6+z = 4z$, $3z = 6$, $z = 2$

$\boxed{z=2,\ x=4}$

only other constraint: $x^2+y^2+z^2 = 36$

$y^2 = 36 - 4^2 - 2^2 = 16$

$y = \pm 4$

2 possibilities:

$\left.\begin{aligned} &\nabla f = k \nabla g \\ &g(x,y,z) = 36 \end{aligned}\right\}$ when (x,y,z) is $(4,+4,2)$, $(4,-4,2)$

(Don't do 2nd deriv)

$$f(4,4,2) = 6x + y^2 + xz \Big|_{(4,4,2)}$$

$$= 6\cdot 4 + 4^2 + 4\cdot 2$$

$$= 24 + 16 + 8 = 48$$

$$f(4,-4,2) = 6\cdot 4 + (-4)^2 + 4\cdot 2 = 48$$

$y=0$:

$f_x = k g_x$

$f_y = k g_y$

$f_z = k g_z$

$x^2+y^2+z^2=36$

$y=0$

$x^2+z^2=36$

$y=0$

Next time...