Math 200, Oct 21

Saddle ??? New to 2-d 3-d

=

Max & Min

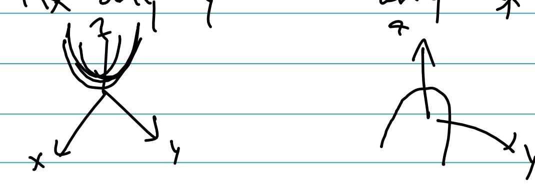
=

$z = f(x,y) = x^2 - y^2$

$f_{xx} = (f_x)_x = (2x)_x = 2$

$f_{yy} = -2$

Fix any y   fix any x



- Fine print with max/min
- New saddle:

sneaky!

$z = f(x,y) = xy$

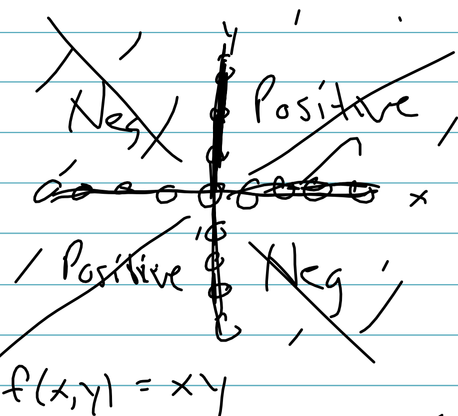
$f_x = y \qquad f_{xx} = 0$

$f_y = x \qquad f_{yy} = 0$

at 0, $\nabla f|_{\text{at } 0,0} = (f_x, f_y)_{\text{at } (0,0)}$

$= (0,0)$.

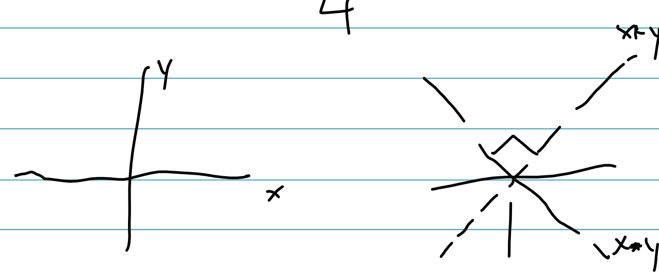
$\nabla f|_{\text{a point}} = (0,0)$ "critical point"



$f(x,y) = xy$

Turns out:

$$xy = \frac{\underbrace{(x+y)^2}_{\text{pos square}} - \underbrace{(x-y)^2}_{\text{neg square}}}{4}$$



Test!

Take

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{xx} \end{vmatrix} = f_{xx}f_{yy} - (f_{xy})^2$$

=

e.g. $f = xy \qquad f_x = y$

$f_{xx} = 0 = f_{yy} \qquad f_{xy} = 1$

$$D = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

When $D < 0$ always saddle

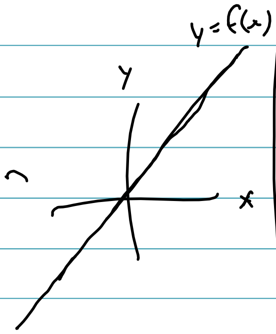
$D = 0$ degenerate / indeterminate

really: we can't tell

2^{nd} fine print:

$y = f(x)$:

max of $f(x) = x$ on the reals



$\underbrace{f'(x)}_{1} = 0$

$1 \neq 0$

1^{st} fine print:

If $\nabla f = (0,0)$ at a point, then

(1) $D = f_{xx} f_{yy} - (f_{xy})^2 < 0$

$\rightarrow$ saddle

$D = f_{xx} f_{yy} - (f_{xy})^2 > 0$

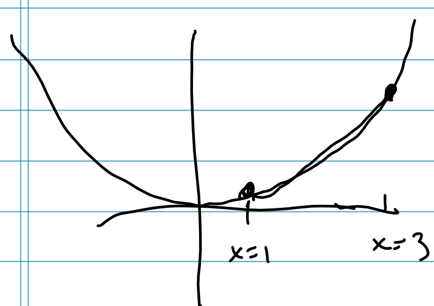
(2a) if $f_{xx} > 0$ (and $D > 0$) local min.

(2b) .. $f_{xx} < 0$ (and $D > 0$) local max

If $D = 0$ not sure

But if $f(x)$ on

$x \in [1,3]$, $f(x) = x^2$



Here $f_x = 2x$

for $x \in [1,3]$, f_x never 0

If we are looking $x \in [1,3]$

x bounded, we have to look at "boundary" $x=1$ and $x=3$

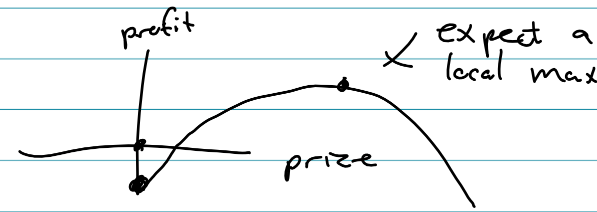
If you have a nice continuous function on the reals

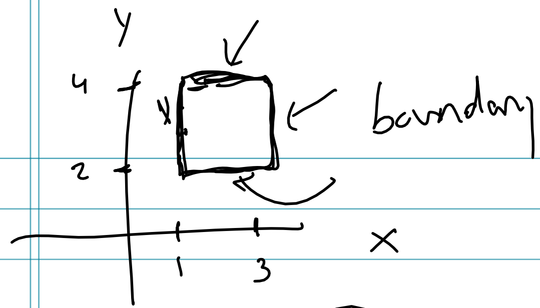
$\{ f(x) \mid x \in \mathbb{R} \}$

or

$\{ f(x,y) \mid x,y \in \mathbb{R} \}$

can't always expect max, min values...





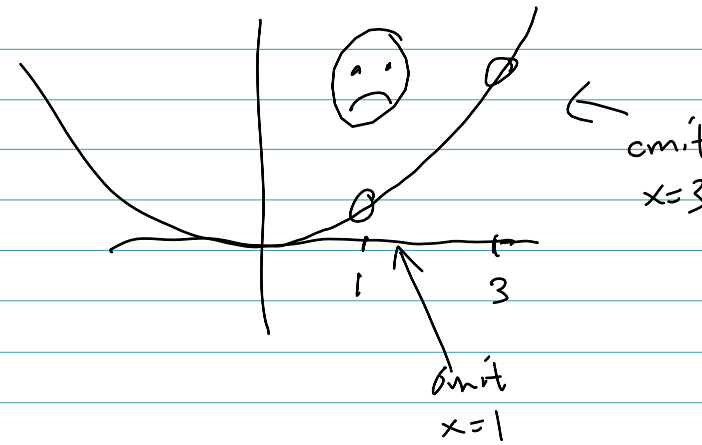
$f(x,y)$: $1 \le x \le 3$

$2 \le y \le 4$

1. min, max expect to be attained somewhere
2. can be along the boundary
3. in the interior: $\nabla f = 0$

   f_{xx}, $D = f_{xx}f_{yy} - (f_{xy})^2 \cdots$

If $f(x) = x^2$ on $(1,3)$



For $f(x)$, $f(x,y)$, $f(x,y,z)$, --

look at min max on

— closed intervals $x \in [1,3]$

Q: Find absolute min/max of $f(x,y) = 5 + 2x - x^2 - 4y^2$ on $-1 \le x \le 3$, $-1 \le y \le 1$

1. Find where $\nabla f = 0$
2. Look at f along boundary

1. $\nabla f = (f_x, f_y)$

   $f_x = 2 - 2x \qquad f_y = -8y$

   $f_x = f_y = 0 \Rightarrow \begin{array}{l} 2 - 2x = 0 \\ -8y = 0 \end{array}$

min/max $f(x,y)$ in here