__Implicit differentiation:__

__Rem:__ $f$ vs $F$ ? (In textbook)

$f$ function explicit (mostly)

$F$ tends to be implicit

Which is better? --

$4x+5y+6z=0$ ← implicit $F(x,y,z)=0$

$\Leftrightarrow$ $-6z=4x+5y$

$$z=\frac{4x+5y}{-6}$$

explicit $z=f(x,y)$

$F(x,y,z)=0$ implicit

$\sin(yz)+3z^3+x+z^{20}(y-2)$

$+e^y=0$

$z=$ no general method

implicit: $x^2+y^2+z^2=17$

at $(x_0,y_0,z_0)=(2,3,-2)$

$$z=-\sqrt{17-x^2-y^2}$$

Tangent plane at $(2,3,-2)$

$$F(x,y,z)=x^2+y^2+z^2-17=0$$

$$F_x=2x,\quad F_y=2y,\quad F_z=2z$$

If $\Delta F=0$, then roughly, at $(2,3,-2)$!

$$F_x(2,3,-2)\,\Delta x+F_y(2,3,-2)\,\Delta y$$
$$+F_y(2,3,-2)\,\Delta z=0$$

linear approx $\Leftrightarrow$ tangent plane

$(x,y,z)$ near $(2,3,-2)$:

$$F_x(2,3,-2)=2x\Big|_{(2,3,-2)}=4$$

$$F_y=2y\Big|_{\text{same point}}=6,\quad F_z=2z=-4$$

__One view tangent plane:__

$$\boxed{4\,\Delta x+6\,\Delta y+(-4)\,\Delta z=0}$$

or

$$4(x-2)+6(y-3)+(-4)(z-(-2))=0$$

explicit:

$$z=-\sqrt{17-x^2-y^2}$$

$$\Delta z=(z_x)\,\Delta x+(z_y)\,\Delta y$$

$$4\Delta x + 6\Delta y + (-4)\Delta z = 0$$

$$\underbrace{\Delta z = \frac{4}{-(-4)}\Delta x + \frac{6}{-(-4)}\Delta y}$$

$z = z_0 +$ this

$$F_x \Delta x + F_y \Delta y + F_z \Delta z = 0$$

$$\iff \Delta z = z_x \Delta x + z_y \Delta y$$

$$\boxed{z_x = \frac{-F_x}{F_z}} \quad = \left(\frac{F_x}{-F_z}\right)\Delta x + \left(\frac{F_y}{-F_z}\right)\Delta y$$

$$z_x = \left(-\sqrt{17 - x^2 - y^2}\right)_x$$

$$= \left(-(17 - x^2 - y^2)^{1/2}\right)_x$$

$$= -\tfrac{1}{2}(17 - x^2 - y^2)^{-1/2}(-2x)$$

$=$

$$F(x,y,z) = 0 = z^9 + 3z^2 y + x + \cos(z)$$

$z =$ explicitly $=$

($F$ to warn us of implicit equs)

$z_x = 1$ ??

if $f'(x_0) \neq 0$

then we can move $\Delta x$

and locally increase $f$

- - . decrease $f$ . .

=

$z = f(x,y)$

if $f_x(x_0, y_0) \neq 0$

then $f$ can be increased

- — decreased

near $(x_0, y_0)$.

Same for $f_y(x_0, y_0) \neq 0$.

14.7

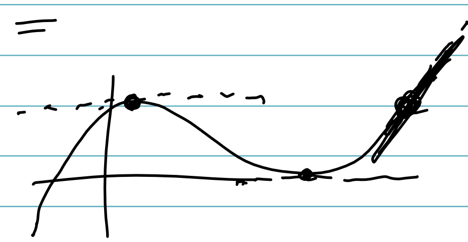
Maxima & minima

same idea (1 variable),

however 2-dim geom

3-dim geom - -

=



$y = f(x)$

if $(x_0, y_0)$ is a local max

$f'(x_0) = 0$

$f(x)$ near $x_0$

$\approx f(x_0) + \Delta x \, f_x(x_0)$

$\approx f(x_0) + (x-x_0) f_x(x_0)$

better approx by

$$f(x_0) + (x-x_0) f_x(x_0) + \frac{(x-x_0)^2}{2} f_{xx}(x_0)$$

if $f_x(x_0)=0$

$$f(x_0) + \frac{(x-x_0)^2}{2} f_{xx}(x_0)$$

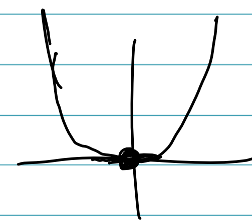
Fact: If $(x_0,y_0)$ is local maximum or local minimum, ~~then~~ for $z = f(x,y)$, then

$$\nabla f(x_0,y_0) = \langle f_x(x_0,y_0), f_y(x_0,y_0) \rangle$$

must be $= \langle 0,0 \rangle$

ex:

$y = x^2$



$y' = 2x$ is $0$ at $x_0=0$

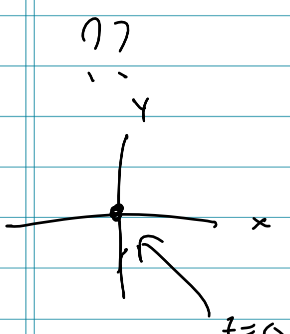
$y''=2>0$ local minimum

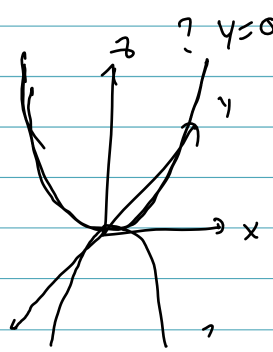
Case 2 ☹ !

$f(x,y) = z = x^2 - y^2$ at $(0,0)$

$f_x = 2x, \quad f_{xy} = 2$

$f_y = -2y, \quad f_{yy} = -2$

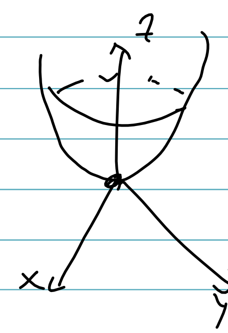



Case 1 !

$z = f(x,y)$ say take

$z = x^2 + y^2$




local min,

$f_x = 2x, \quad f_{xx} = 2$

$f_y = 2y, \quad f_{yy} = 2$

Test works:

test $f_{xx}$

test $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$

$= (f_{xx})(f_{yy}) - (f_{xy})^2$

Claim:

$f_{xx} > 0$ but $D < 0$

~~st~~addle

Saddle