2010 WT1 : Problem 2 :

$T = F(x,y,z)$ = blah blah blah

Directional Derivative = ?

$T = f(x,y)$ simpler (12.1)

$T = F(x)$, $\Delta F = F_x \Delta x$

$= \langle F_x \rangle \cdot \langle \Delta x \rangle$

no geometry . .

$T = F(x,y)$: $\Delta F = \langle F_x, F_y \rangle \langle \Delta x, \Delta y \rangle$

Math 200, Oct 16 :

14.1, 14.3, 14.4 preliminary

14.5 Chain rule

very very similar to 1st year calc.

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14.6 : One idea : $F = F(x,y,z)$

$\Delta F \approx F_x \Delta x + F_y \Delta y + F_z \Delta z$

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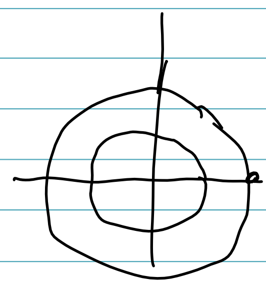
Geometric $= \langle F_x, F_y, F_z \rangle \cdot \langle \Delta x, \Delta y, \Delta z \rangle$

dot product (12.3)



(2,0)

near (2,0)

$\Delta F = \Delta (x^2+y^2)$

$= \langle 2 \cdot 2, 2 \cdot 0 \rangle \cdot \langle \Delta x, \Delta y \rangle$

$= \langle 4, 0 \rangle \cdot \langle \Delta x, \Delta y \rangle$

$= 4 \Delta x$

=

new

$\nabla F = \langle F_x, F_y \rangle$

$T = \text{Temp} = f(x,y) = x^2 + y^2$

ΔF near (x_0, y_0) = ?

$F_x = 2x$, $F_y = 2y$

ΔF near (x_0, y_0)

$\approx F_x(x_0,y_0) \Delta x + F_y(x_0,y_0) \Delta y$

$= \langle F_x, F_y \rangle \cdot \langle \Delta x, \Delta y \rangle$

$= \langle 2x_0, 2y_0 \rangle \cdot \langle \Delta x, \Delta y \rangle$

Direction of greatest increase

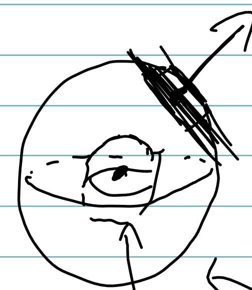
∇F at some point, P, is

$\nabla F = \langle F_x, F_y, F_z \rangle$

Unit normal of ∇F

$\dfrac{(\nabla F)}{\|\nabla F\|}$. "Direction" means "unit vector"

$T = F(x,y,z) = x^2 + y^2 + z^2$



$\nabla F = \langle 2x, 2y, 2z \rangle$

$\langle F_x, F_y, F_z \rangle$

$x^2+y^2+z^2=5$

$x^2+y^2+z^2=1.2$

$\Delta F = \underbrace{\langle 2x, 2y, 2z \rangle}_{\text{vector } (\nabla F)} \cdot \langle \Delta x, \Delta y, \Delta z \rangle$

vector (∇F) • change

$F_z = 2z = 2$

∇F at $(3,2,1)$ is

$= \langle F_x, F_y, F_z \rangle$ at $(3,2,1)$

$= \langle 1, -1, 2 \rangle$

Unit vector : $\dfrac{\langle 1,-1,2 \rangle}{|\langle 1,-1,2 \rangle|}$

$= \left\langle \dfrac{1}{\sqrt{6}}, \dfrac{-1}{\sqrt{6}}, \dfrac{2}{\sqrt{6}} \right\rangle$ greatest increase

$\left\langle \dfrac{-1}{\sqrt{6}}, \dfrac{1}{\sqrt{6}}, \dfrac{-2}{\sqrt{6}} \right\rangle$ " decrease

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2010 WT1 Prob 2

$T = F(x,y,z)$

$= 3 + xy - y^2 + z^2 - x$

At $(x_0, y_0, z_0) = (3,2,1)$

greatest increase } direction
greatest decrease }

$\nabla F = \langle F_x, F_y, F_z \rangle$

$F_x = y - 1$, at $(3,2,1)$ $F_x = 1$

$F_y = x - 2y$ at $(3,2,1)$ $F_y = 3 - 2 \cdot 2 = -1$