14.4, 14.5

Say $f(x,y) = z = x^4 y^3$

Say measure

f at $\boxed{x = 1.713 \quad y = 2.915}$

x, y could be off by 1%

How much can f = z be off by?

$$\boxed{\Delta f = f_x(x_0,y_0)\,\Delta x + f_y(x_0,y_0)\,\Delta y}$$

linear approximation

=

Make easier: $\boxed{(x_0,y_0) = (1,3)}$

$$\boxed{f(x,y) \approx f(x_0,y_0) + f_x(x_0,y_0)\,\Delta x + f_y(x_0,y_0)\,\Delta y}$$

So

$$f(x,y) = f(1,3) + f_x(1,3)\,\Delta x + f_y(1,3)\,\Delta y$$

$\Delta x = x - 1$ how x is altered

$\Delta y = y - 3$ " y " "

$f_x = 4x^3y^3$, $f_x(1,3) = 4 \cdot 27$

$= 108$

$f_y = x^4 3y^2$, $f_y(1,3) = 27$

Approximation near (1,3) is

$$\Delta f \approx 108\,\Delta x + \cancel{108}\,27\,\Delta y$$

$x_0 = 1$, could be off by 1%

$f(x_0 + \Delta x, y_0 + \Delta y)$

$|\Delta x|$ at most 1% of $x = 0.01$

$|\Delta y|$ is off by at most 1%

$y = 3$, $|\Delta y| \le 0.03$

In this case

$$\Delta f \approx 108\,\underbrace{\Delta x}_{\text{at most } 0.01} + 27\,\underbrace{\Delta y}_{\text{at most } 0.03}$$

at most = in absolute value

$$|\Delta f| \le (108)|\Delta x| + 27|\Delta y|$$
$$\le (108)(0.01) + 27(0.03)$$

$x_0 = 1$, Δx

$f(x_0 + \Delta x) = ?$

x_0, x are functions

$x = g(u)$

$f = f(x) = f(g(u))$

(Δf) for a change in u

$= f_x(x_0)\,\Delta x$ say $108\,\Delta x$

We know

$f(1+\Delta x, 3+\Delta y)$

$\approx f(1,3) + 108\,\Delta x + 27\,\Delta y$

What if x, y given by functions of u? u and v? ---

Say $f(1+\Delta x) \approx f(1) + 108\,\Delta x$

(ignore y, Δy ..) (Section 12,1)

Say at $u_0 = 5$

$g(u_0) = g(5) = 1$

At u_0: $x_0 = g(u_0) = g(5) = 1$

we know near $x = x_0 = 1$

$\Delta f \approx 108(\Delta x)$.

Then if $\Delta x \approx (\Delta u)\,g'(5)$

then

$$\Delta f = \underbrace{108}_{f_x}(\underbrace{g'(5)}_{g_u}\,\Delta u) = f_x\, g_u\, \Delta u$$