(y fixed)

$$f_x = z_x = \left(x^2 + 3y^3\right)_x$$

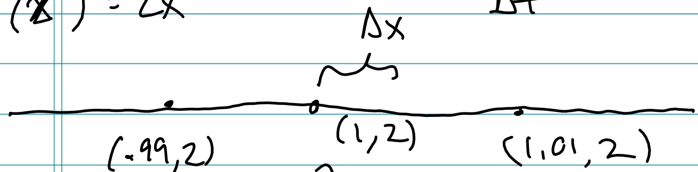
$$= 2x$$

$$f(x+\Delta x, y)$$

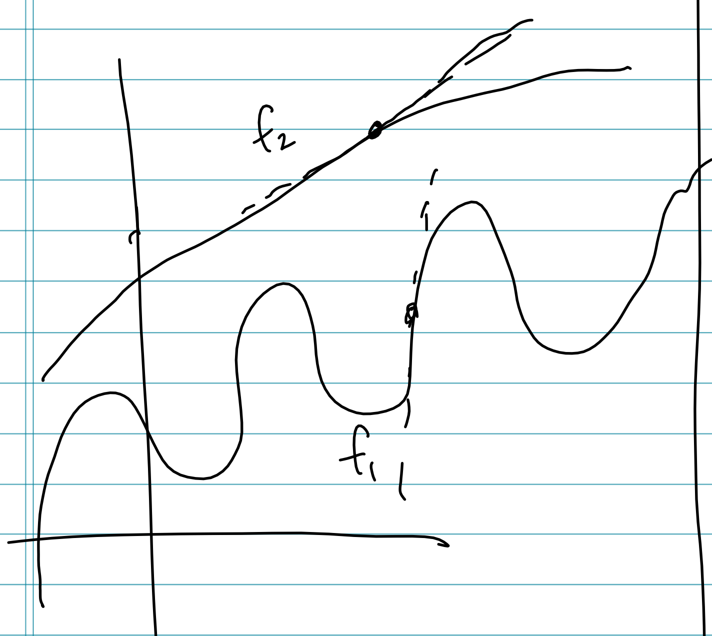
$$= f(x,y) + \Delta x \, \frac{\partial f}{\partial x}(x,y)$$

$(x^2)' = 2x$

Δf



$\Delta(x^2+3y^3)$

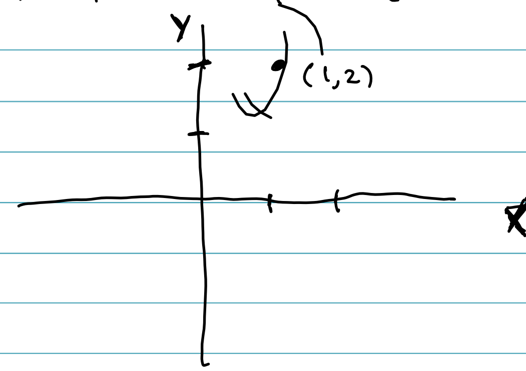


14.4 (revisit 12.4...)

(★) $\boxed{\Delta z \approx f_x(x_0,y_0)\,\Delta x + f_y(x_0,y_0)\,\Delta y}$

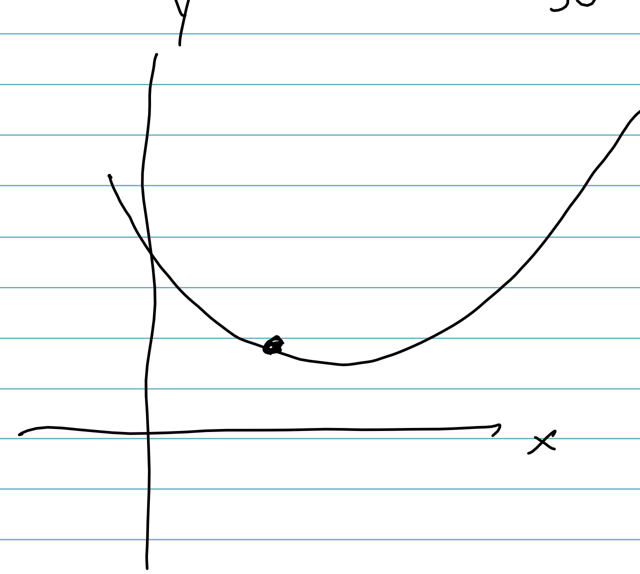
$$z = f(x,y) = x^2 + 3y^3$$

at, say, $(x_0,y_0) = (1,2)$



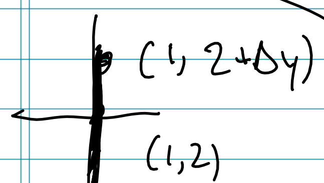
Why is (★) (approximately) true?
How to analyze change/error?

30 C



$$f(1+\Delta x, 2) \approx f(1,2) + \Delta x \cdot 2$$

$$f(1, 2+\Delta y) = f(1,2) + \Delta y \left(\text{deriv of } f \text{ holding } x \text{ fixed varying } y\right)$$



$f = x^2 + 3y^3$, $f_y = 9y^2$

Δz or Δf

$$f(1+\Delta x, 2+\Delta y) - f(1,2)$$

$$= 2\,\Delta x + 36\,\Delta y$$

$$= f_x(1,2)\,\Delta x + f_y(1,2)\,\Delta y$$

=

Cross product:

small Δx: $\Delta f = \Delta z$
$= 2\,\Delta x$

tangent contains $\langle 1, 0, 2 \rangle$

$$(x_0, y_0) = (1,2)$$

$$f_x(x_0, y_0) = (x^2 + 3y^3)_x \text{ at } x = x_0,\ y = y_0$$

$$= (2x) \text{ at } x = x_0 = 1,\ y = y_0 = 2$$

$$= 2$$

$$f(1+\Delta x, 2) = f(1,2) + 2\Delta x$$

should be valid also near $(1,2)$

at $(x_0, y_0) = (1,2)$

$$f_y = 9y^2 = 9 \cdot 2^2 = 36$$

$$f(1+\underline{\Delta x}, 2) = f(1,2) + \underline{\Delta x \cdot 2}$$

$$f(1, 2+\underline{\Delta y}) = f(1,2) + \underline{\Delta y \cdot 36}$$

$$\boxed{f(1+\Delta x, 2+\Delta y) = f(1,2) + 2\Delta x + 36\Delta y}$$

$$\left\langle \begin{vmatrix} 0 & 2 \\ 1 & 36 \end{vmatrix}, -\begin{vmatrix} 1 & 2 \\ 0 & 36 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\rangle$$

$$= \langle -2, -36, 1 \rangle$$

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

goes through $\langle x_0, y_0, z_0 \rangle$

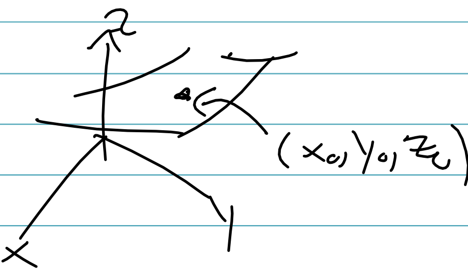
normal direction

$\langle a, b, c \rangle$

$$\Delta z = 36\,\Delta y$$

$$\langle 0, 1, 36 \rangle$$

$\Delta x \quad \Delta y \quad \Delta z$



Normal to

$$\langle 1, 0, 2 \rangle \times \langle 0, 1, 36 \rangle$$

$$-2(x-x_0) - 36(y-y_0) + 1(z-z_0) = 0$$

$$z - z_0 = 2(x-x_0) + 36(y-y_0)$$

$$= f_x(x-x_0) + f_y(y-y_0)$$