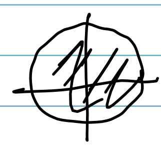
14.1 What's the domain of

(1) $z = \sqrt{9 - x^2 - y^2}$

Ans: $9 - x^2 - y^2 \geq 0$

$9 \geq x^2 + y^2$



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(2) $z = \sqrt{x^2 + 3\sin(y) x^9 - 12.1 x^5 - xy^{11} - \ldots}$

- No homework assigned this week
- Midterm Oct 9 (Friday)
  on 12.1 - 12.5, a bit of 12.6
  (detailed list by Friday)

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14.1 - Domain, Level Sets

$z = f(x, y)$

~~14.2 - Limits & Continuity skipped~~

14.3 - Patrial Derivatives

- Clairaut's Thm: $f_{xy} = f_{yx}$

$\boxed{\text{14.4 - Tangent Planes \& Linear Approx}}$

(12.1) If 2-d or 3-d is difficult, try 1-d

We know 1-dim calculus

$(fg)' = fg' + f'g$

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In one varible:

$(10 \cdot f(x))' = 10 \cdot f'(x)$

=

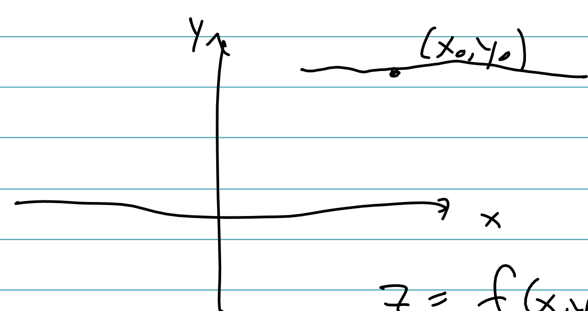
$f(x)' = 0$ then

$f(x) = \text{constant} \ldots$

=

$f = f(x,y)$ and $f_x = 0 \ldots$

14.3: Patrial Derivatives



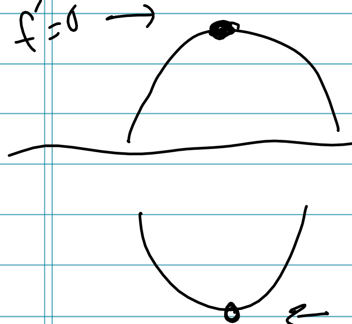
$z = f(x,y)$

How does z change?

$f_x(x_0, y_0) =$ diff $f(x, y_0)$ in x
hold $y = y_0$

$(fg)_x = f\, g_x + f_x g$

14.3 Higher Partial Derivatives



$\Leftrightarrow f'' < 0$

$\Leftrightarrow f'' > 0$

$$f_{xx} = (f_x)_x = \frac{\partial^2}{\partial x^2} f$$

$$f_{xy} = (f_x)_y = \frac{\partial}{\partial y}\frac{\partial}{\partial x} f$$

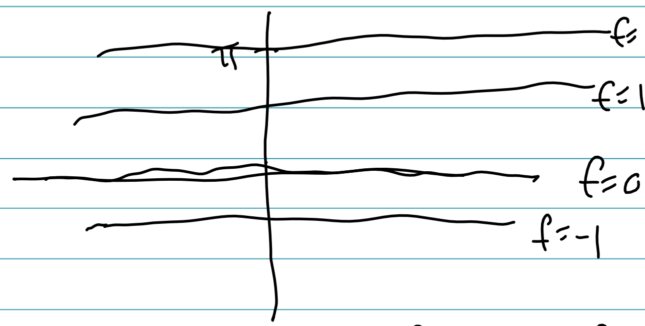
$$f_{yx},\ f_{yy}$$

$$\begin{tikzcd} & x^3 y^4 \arrow[dl, "\frac{\partial}{\partial x}"'] \arrow[dr, "\frac{\partial}{\partial y}"] & \\ 3x^2 y^4 \arrow[dr, "\frac{\partial}{\partial y}"'] & & \cancel{3}x^3 4y^3 \arrow[dl, "\frac{\partial}{\partial x}"] \\ & 3x^2 4y^3 & \end{tikzcd}$$

What if $f_x(x_0, y_0) = 0$ for all x_0, y_0?

Constant in x ...

$$(\sin(y))_x = \frac{\partial}{\partial x}\sin(y) = 0$$



$f_x = 0$ then $f(x,y) = f(y)$

Clairaut's Thm: $f_{xy} = f_{yx}$

Eg. $f = x^3 y^4$

$f_x = 3x^2 y^4$, $f_y = x^3 4y^3$

$(f_x)_y = 3x^2 4y^3$, $(f_y)_x = 3x^2 4y^3$

29 A

$f(x,y) =$

$x^2 \sin(y) + x^3 y^4$

$f_{xy} = \left(x^2 \sin(y) + x^3 y^4\right)_{xy}$

$= \left(x^2 \sin(y)\right)_{xy} + \left(x^3 y^4\right)_{xy}$

Claim! $f = g + h$

$f_{xy} = g_{xy} + h_{xy}$

Why? $(g+h)_x = g_x + h_x$

$(g+h)_{xy} = (g_x + h_x)_y$

Taylor Series!

$f(\cancel{x_0})$ near $x = x_0$

$f(x) = f(x_0) + f'(x_0)(x-x_0) + f''(x_0)\frac{(x-x_0)^2}{2}$

$+ \dots$

stop here

polynomial in x of degree 2

$$\begin{tikzcd}
 & x^2 \sin(y) \arrow[dl, "\frac{\partial}{\partial x}"'] \arrow[dr, "\frac{\partial}{\partial y}"] & \\
2x \sin(y) \arrow[d, "\frac{\partial}{\partial y}"'] & & x^2 \cos(y) \arrow[d, "\frac{\partial}{\partial x}"] \\
2x \cos(y) & = & 2x \cos(y)
\end{tikzcd}$$

29 C

$f = x^2 \sin(y) + x^3 y^4$

$f_{xy} = (\quad)_{xy} + (\quad)_{xy}$

$= (\quad)_{yx} + (\quad)_{yx}$

$= f_{yx}$