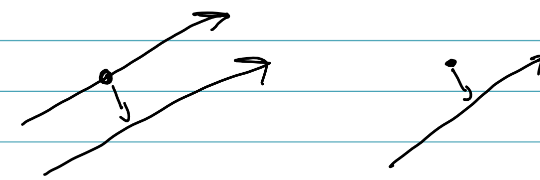
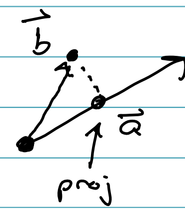
Review 12.3



Distance ~~of~~ ① 2 parallel lines

② a point and a line



distance $= |\vec{b} - \overline{\text{proj}}|$

$= |\vec{b} - \text{proj}_{\vec{a}} \vec{b}|$

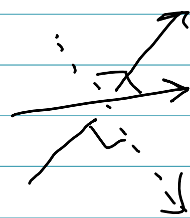
$= \ldots$

... Part of toolbox -- Like integration

Many 3-dim ideas analogous to 2-dim ideas.

An exception!! :

2 ~~s~~kew lines



Say!

L_1: $\langle x,y,z \rangle = \underbrace{\langle 1,2,2 \rangle}_{\text{point}} + t\underbrace{\langle 1,0,3 \rangle}_{\text{direction}}$

L_2: $\langle x,y,z \rangle = \langle 2,5,1 \rangle + t\langle 2,0,-1 \rangle$

$\langle 1,0,3 \rangle \times \langle 2,0,-1 \rangle =$

$\left\langle 0, -\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}, 0 \right\rangle$

$= \langle 0, 7, 0 \rangle$

Line L_1 should be on plane:

$0 \cdot x + 7 \cdot y + 0 \cdot z + d_1 = 0$

point L_1: $\langle 1,2,2 \rangle$ is on L_1

step in direction $\langle 1,0,3 \rangle$

we had better stay on

$0 \cdot x + 7 \cdot y + 0 \cdot z + d_1 = 0$

☺ always true :

$\langle 0,7,0 \rangle \perp \langle 1,0,3 \rangle$

~~$d_1 = \langle 0 \ldots$~~

d_1 given by

$0 \cdot x_0 + 7 \cdot y_0 + 0 \cdot z_0 + d_1 = 0$

for any point $\langle x_0, y_0, z_0 \rangle$ on L_1 :

$0 \cdot 1 + 7 \cdot 2 + 0 \cdot 2 + d_1 = 0$

$14 + d_1 = 0 \qquad d_1 = -14$

22A

L_1 lies on the plane

$$0\cdot x + 7\cdot y + 0\cdot z + (-14) = 0$$

L_2: $(2,5,1) + t\langle 2,0,-1\rangle$

$$0\cdot 2 + 7\cdot 5 + 0\cdot 1 + d_2 = 0$$

$35 + d_2 = 0 \quad d_2 = -35$

$$0\cdot x + 7\cdot y + 0\cdot z + (-35) = 0$$

$$\frac{|d_1 - d_2|}{|\langle 0,7,0\rangle|} = \frac{|-14-(-35)|}{7} = 3$$

22B

other L_1 direction $\langle 1,2,5\rangle$

other L_2 " $\langle -3,7,8\rangle$

$$\langle 1,2,5\rangle \times \langle -3,7,8\rangle$$

=

What is 12.6 ???

Surfaces of degree 2 in 3-dimensions...

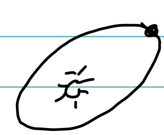
$$5x^2 + 2xy + 3y^2 + 4xz - 21yz + 3z^2 - 2x + 5y + 7x + 12z = 0$$

22C

10.5: Quadratic equations in 2-dims: parabola, ellipse, hyperbola

=

Sun & earth: ellipse 10.5



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Earth: ellipsoid

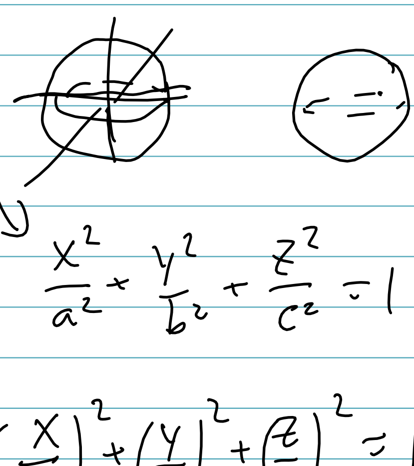
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

22D

$$x^2 + y^2 = 1$$

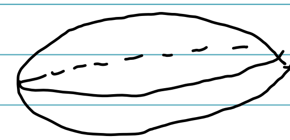
$$x^2 + y^2 + z^2 = 1$$

(12.1)



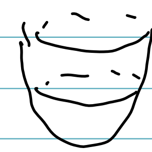
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$



(p.stachio)

Satellite dish

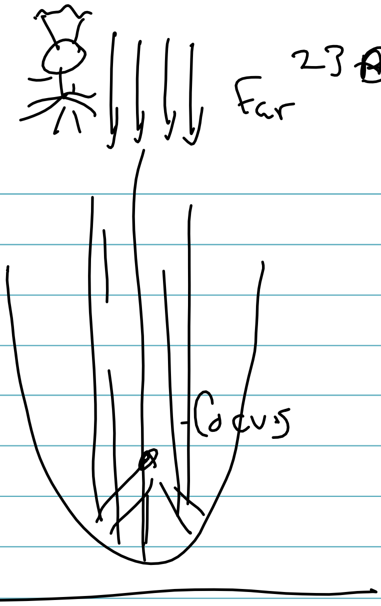


parabola

$(4p)z = x^2$

x ↗ y ↘ (if y not in picture)

$(4p)(z) = (x^2+y^2)$



A – G table #1

H – L table #2

M – S chairs →

T – Z chairs →