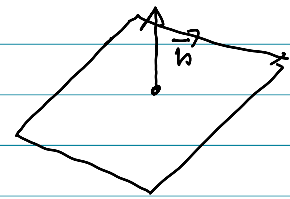
Review eqns for plane

all points x, y, z with

$$\boxed{ax + by + cz + d = 0}$$

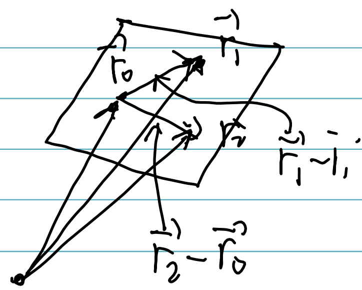
$$\langle a, b, c\rangle \cdot \langle x, y, z\rangle + d = 0$$

$\vec{n} = \langle a, b, c\rangle$ is the normal vector



- If we know $\vec{n} = \langle a, b, c\rangle$ and a single point $\langle x_0\ y_0\ z_0\rangle$ we can find $d = -\vec{n} \cdot \langle x_0\ y_0\ z_0\rangle$
  $\rightsquigarrow$ get eqn.

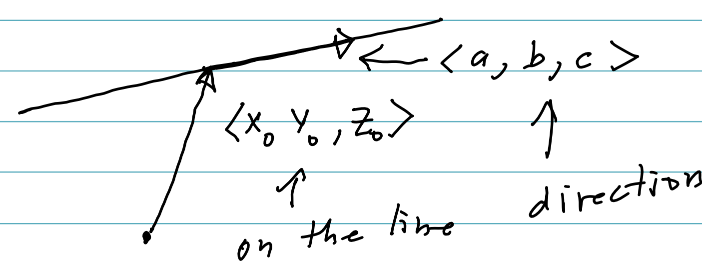
- if we know 3 pts $\vec{r}_0\ \vec{r}_1\ \vec{r}_2$ on the plane



$$\vec{n} = (\vec{r}_2 - \vec{r}_0) \times (\vec{r}_1 - \vec{r}_0)$$

$\rightsquigarrow$ get eqn.

Lines in 3d



points on the line are

$$\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$$

for every $t \in \mathbb{R}$.

parametric form

can write as

$$x = x_0 + ta,\quad y = y_0 + tb,\quad z = z_0 + tc$$

t is same so

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Example: find the intersection of

$$x + 2y + z + 4 = 0$$
$$2x + 3y - z = 0$$

$\rightsquigarrow$ can solve using Gaussian elimination (omit)

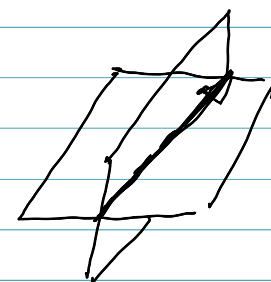
can also use cross prod.

are the planes parallel?

19A

check normal vectors

$\vec{n}_1 = \langle 1, 2, 1 \rangle$
$\vec{n}_2 = \langle 2, 3, -1 \rangle$ not $\parallel$



direction vector for line is $\perp$ to both $\vec{n}_1$, $\vec{n}_2$ since it lies in both planes.

$$\text{direction vector} = \vec{n}_1 \times \vec{n}_2$$
$$= \langle -5, 3, -1 \rangle$$

Need a single point on line

19B

try to find a soln to the eqns where $z = 0$

$$\left.\begin{aligned} x + 2y &= -4 \\ 2x + 3y &= 0 \end{aligned}\right\} \text{soln } y = -8,\ x = 12$$

so $\langle 12, -8, 0 \rangle$ lies on the line $\rightsquigarrow$ eqn is

$$\boxed{\langle x, y, z \rangle = \langle 12, -8, 0 \rangle + t \langle -5, 3, -1 \rangle}$$

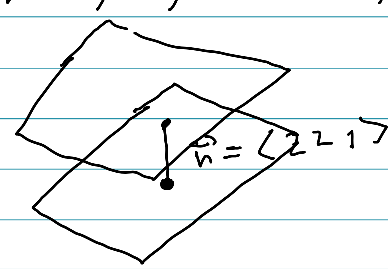
Example 2: distance between $\parallel$ planes.

19C

Find distance between

P_1: $\langle 2, 2, 1 \rangle \cdot \langle x, y, z \rangle + 9 = 0$

P_2: $\langle 2, 2, 1 \rangle \cdot \langle x, y, z \rangle - 18 = 0$



Suppose $\langle x_0, y_0, z_0 \rangle$ lies on P_1

Find α so that

$$\langle x_0, y_0, z_0 \rangle + \alpha \langle 2, 2, 1 \rangle$$

lies on second plane

Then $\| \alpha \langle 2, 2, 1 \rangle \|$ is the distance

19D

- $\langle 2, 2, 1 \rangle \cdot \langle x_0, y_0, z_0 \rangle = -9$
- $\langle 2, 2, 1 \rangle \cdot \left(\langle x_0, y_0, z_0 \rangle + \alpha \langle 2, 2, 1 \rangle \right) = 18$

$$-9 + \alpha \left(\langle 2, 2, 1 \rangle \cdot \langle 2, 2, 1 \rangle \right) = 18$$

$$\alpha \cdot (4 + 4 + 1) = 27$$

$$\alpha = 3$$

$$\text{dist} = \| \alpha \langle 2, 2, 1 \rangle \|$$
$$= 3 \sqrt{4 + 4 + 1} = 9$$

21B

21A

Example 3 Find the distance between 2 skew lines in 3 d

- let $\vec{n}$ be $\perp$ to the direction vector of both lines. (find using cross prod)
- Then the lines lie on parallel planes with direction vector $\vec{n}$.
- dist between lines = dist between those planes.

21D

21C