15A

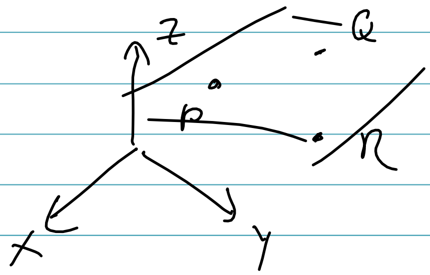
$\vec{v} \to \langle 1,3\rangle$

$\langle -3,1\rangle \cdot \langle x,y\rangle = \text{something}$

e.g. $\langle -3,1\rangle \cdot \langle 2,1\rangle = \text{something}$

$-6+1 = -5$ !

=




$ax+by+cz = \text{something}$

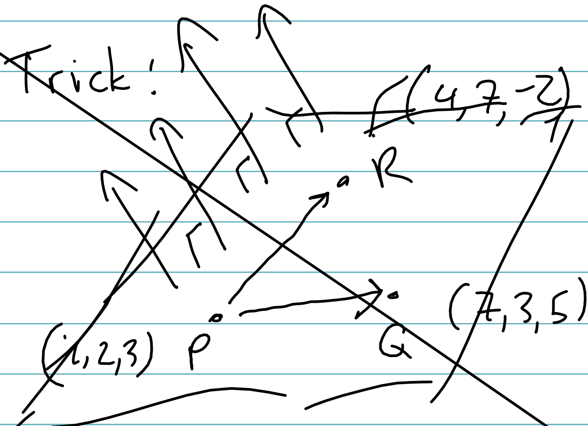
15B

$$\begin{cases} a\cdot 1 + b\cdot 2 + c\cdot 3 + d = 0 \\ a\cdot 4 + b\cdot 7 + c(-2) + d = 0 \\ a\cdot 7 + b\cdot 3 + c\cdot 5 + d = 0 \end{cases}$$

$$\begin{bmatrix} \ \\ - \\ \ \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} -$$

15C

Trick!




~~Take~~

$$\boxed{\langle a,b,c\rangle = \overrightarrow{PQ} \times \overrightarrow{PR}}$$

$\langle a,b,c\rangle \cdot \langle x,y,z\rangle = \text{something}$

Case something $= 0$:

$\langle a,b,c\rangle \cdot \langle x,y,z\rangle = 0$

$\langle a,b,c\rangle \perp \langle x,y,z\rangle$

15D

Upshot:

$\langle a,b,c\rangle =$

$\langle 6,1,2\rangle \times \langle 3,5,-5\rangle$

= blah..

$= \left\langle \cancel{\begin{vmatrix} 1 & 2 \\ 5 & -5 \end{vmatrix}}, -\begin{vmatrix} 6 & 2 \\ 3 & -5 \end{vmatrix}, \right\rangle$

$\begin{vmatrix} 6 & 1 \\ 3 & 5 \end{vmatrix}$

6 1 2
3 5 -5

Trick: $\vec{v}, \vec{u}$ then $\vec{v} \times \vec{u}$ is orthogonal to $\vec{v}$ and $\vec{u}$

12.5:

Line $\leftrightarrow$ 2 equations

Plane $\leftrightarrow$ 1 equation

=

$$\langle a,b,c\rangle = \langle -5-10,\ -\overset{-36}{\left(6(-5)-2\cdot 3\right)},\ 6\cdot 5-3\cdot 1\rangle$$

$$= \langle -15, 36, 27\rangle$$

$\langle -15,36,27\rangle \cdot \langle x,y,z\rangle =$ something

To find "something"

$$\langle -15,36,27\rangle \cdot \langle 1,2,3\rangle = \underset{153}{-15+72+81} = 138$$



$$-15x+36y+27z = 138$$

$$-15x+36y+27z-138=0$$

=

$$\underbrace{\langle -15,36,27\rangle}_{\vec{n}} \cdot \langle 7,3,5\rangle$$

$$= 138$$

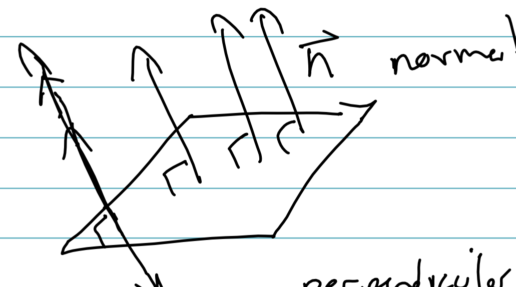
(Exercise)

=



Plane:

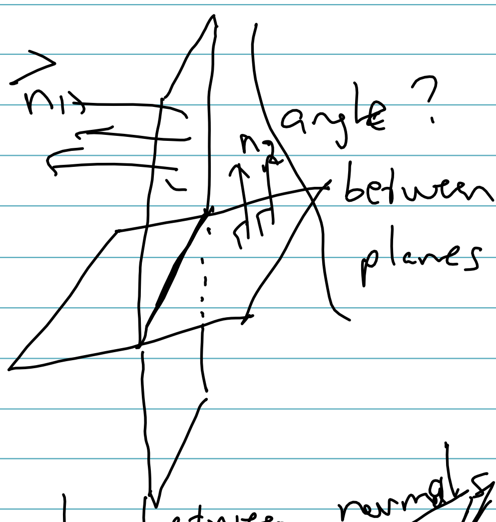
$\langle a,b,c\rangle \langle x,y,z\rangle =$ something



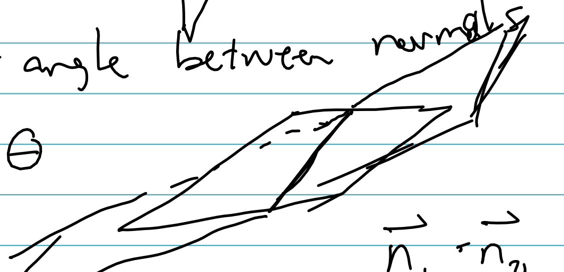

$\langle a,b,c\rangle$ is the perpendicular / othrogonal / normal direction

$\vec{n}$ determined up to scaler






= angle between normals

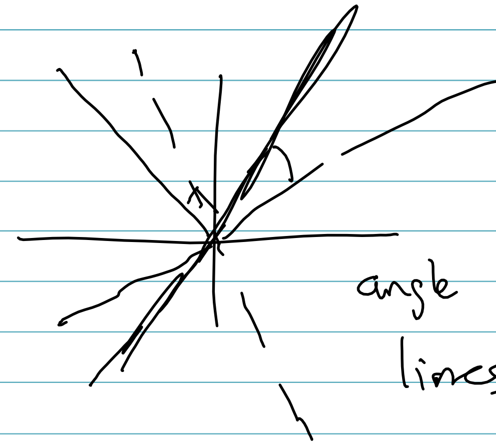


$= \theta$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|}$$

17B

17A



angle between lines

= angle between normals

17D

17C