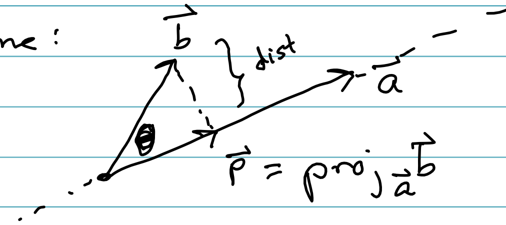
9A

12.3

Last time:



$\vec{p} = \text{proj}_{\vec{a}}\vec{b}$

$$\text{dist} = |\vec{b} - \vec{p}| = |\vec{b}|\,|\sin\theta|$$

$$|\vec{p}| = |\vec{b}|\,|\cos\theta|$$

In 3-dim

$$\langle b_1, b_2, b_3\rangle \times \langle c_1, c_2, c_3\rangle$$

$$= \left\langle \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix}, -\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix}, \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \right\rangle$$

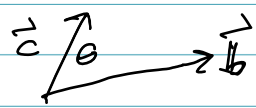
9B

Last Time: $\vec{b} \times \vec{c}$ $\perp$ to $\vec{b}$

$\perp$ to $\vec{c}$

and

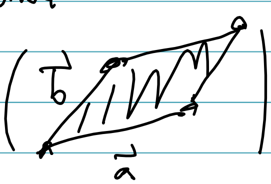
$$|\vec{b} \times \vec{c}| = |\vec{b}|\,|\vec{c}|\,|\sin\theta|$$



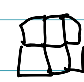
in 3-dim

$$\text{Det}(\vec{a}, \vec{b}) = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

in 2-dim

area  $= \pm\,\text{Det}(\vec{a}, \vec{b})$

9C

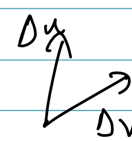


$\rightsquigarrow$

$u = f(x, y)$

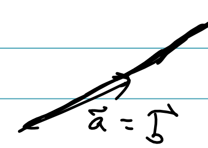
$v = g(x, y)$

Δv!



$\Delta x, \Delta y$

$\text{Det}(\vec{a}, \vec{b})$: $\vec{a} = \vec{b}$



area $= 0$

9D

10A

=

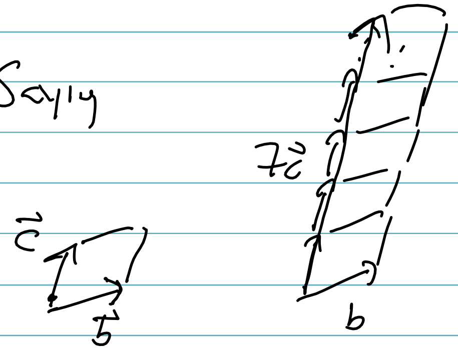
$\vec b \times (7\vec c) = \left\langle \begin{vmatrix} b_2 & b_3 \\ 7c_2 & 7c_3 \end{vmatrix}, \dots \right\rangle$

$\begin{vmatrix} b_2 & b_3 \\ 7c_2 & 7c_3 \end{vmatrix} = 7(b_2c_3 - b_3c_2) = 7\left|\ \right|$

10B

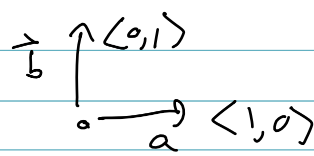
$\vec b \times (7\vec c) = 7(\vec b \times \vec c)$

Sayly



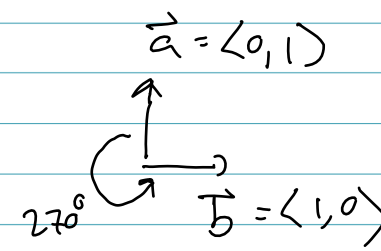
=

$Det(\vec a, \vec b) = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$



$Det = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

10C



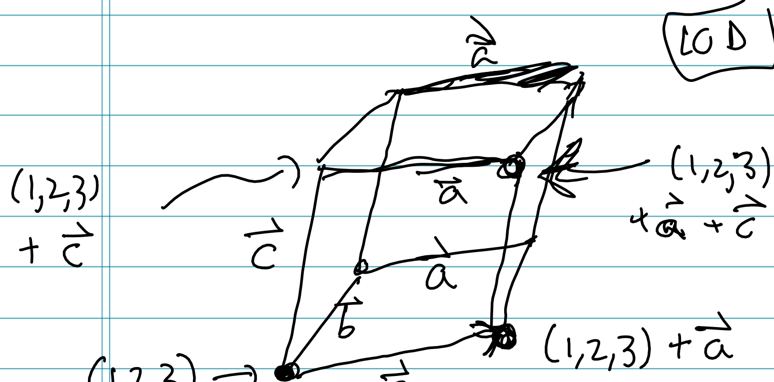
$Det = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

=

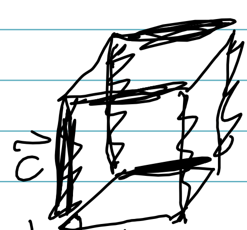
$Det(\vec a, \vec b, \vec c)$

$= a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + \dots$

10D



Parallelopiped with sides $\vec a, \vec b, \vec c$

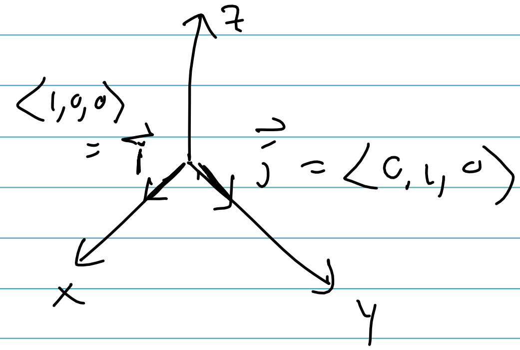


Volume(Parallelopiped)
$= \pm Det(\vec a, \vec b, \vec c)$

13A

$\vec{b} \times \vec{c}$

$\langle 1,0,0\rangle \times \langle 0,1,0\rangle$



$\vec{i} \times \vec{j} = \langle 1,0,0\rangle \times \langle 0,1,0\rangle$

$= \left\langle 0,\ 0,\ \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\rangle$

13B

$= \langle 0,0,1\rangle = \vec{k}$

$\vec{i} \times \vec{j} = \vec{k}$

Right-handed rule:

$\vec{b} \times \vec{c} \perp$ to $\vec{b}, \vec{c}$

and

$|\vec{b} \times \vec{c}| = |\vec{b}|\,|\vec{c}|\,(\sin\theta)$

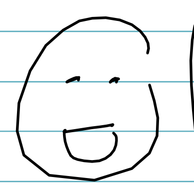
13C

$\text{Det}(\vec{a}, \vec{b}, \vec{c})$

$= \vec{a} \cdot \left\langle \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix}, -- \right\rangle$

= (Wow!)

$= \vec{a} \cdot (\vec{b} \times \vec{c})$



13D