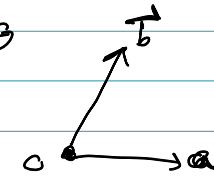
1

Remarks: - Optional Skills Building 1
- Homework due in class on Friday
- Book uses law of cosines to show

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$



- 12.3 "Omit":
  (1) Direction Angles, Direction Cosines
  (2) Scalar projection (but not vector projection)
- Note: Formula Sheet

HW: $|\langle 1,2\rangle|$ how many ops?

2

$$|\langle 1,2\rangle| = \sqrt{1\cdot 1 + 2\cdot 2}$$

1 square root, 2 mults, 1 add

Unit vector in direction $\langle 1,2\rangle$:

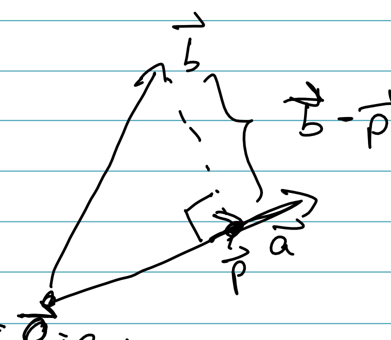
$$\frac{\langle 1,2\rangle}{|\langle 1,2\rangle|} = \left\langle \frac{1}{|\langle 1,2\rangle|}, \frac{2}{|\langle 1,2\rangle|} \right\rangle$$

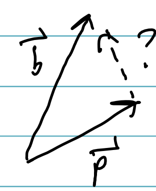
$\boxed{\text{4 operations. Remember}}$

division, division

3

12.3:





$\vec{p} + (?) = \vec{b}$

$? = \vec{b} - \vec{p}$

(1) $\vec{p} = \alpha\vec{a}$

(2) $(\vec{b} - \vec{p}) \perp \vec{a}$

4

$$(\vec{b} - \vec{p}) \cdot \vec{a} = 0$$

$$\vec{b}\cdot\vec{a} - \vec{p}\cdot\vec{a} = 0$$

$$\vec{b}\cdot\vec{a} = \vec{p}\cdot\vec{a}$$

$$= \alpha\,\vec{a}\cdot\vec{a}$$

$$\boxed{\alpha = \frac{\vec{b}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}}$$

$$\vec{p} = \mathrm{proj}_{\vec{a}}\vec{b} = \alpha\vec{a}$$

$$= \vec{a}\left(\frac{\vec{b}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\right)$$

5

E.g.: $\vec{a} = \langle 1,1,1 \rangle$

$\vec{b} = \langle 2.00, 2.04, 2.05 \rangle$

words frogs in BC, '' AB, '' ON

$\text{proj}_{\vec{a}} \vec{b}$ = what multiple of $\langle 1,1,1 \rangle$ is closest to $\vec{b}$ = experimental data $= \langle 2.00, \dots$

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$$\text{proj}_{\vec{a}} \vec{b} = \vec{a} \left(\frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}} \right)$$

$$= \langle 1,1,1 \rangle \frac{\langle 1,1,1 \rangle \cdot \langle 2.00, 2.04, 2.05 \rangle}{\langle 1,1,1 \rangle \cdot \langle 1,1,1 \rangle}$$

$$= \langle 1,1,1 \rangle \frac{2.00 + 2.04 + 2.05}{1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1}$$

$$= \langle 1,1,1 \rangle \frac{2.00 + 2.04 + 2.05}{3}$$

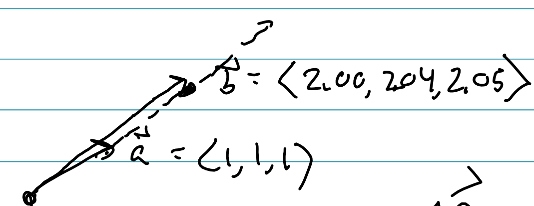
$$= \langle 1,1,1 \rangle \, \text{Average}(2.00, 2.04, 2.05)$$

$$= \langle 1,1,1 \rangle \, 2.03$$

$$= \langle 2.03, 2.03, 2.03 \rangle$$

7

$$\vec{b} = \text{proj}_{\vec{a}} \vec{b} + (\vec{b} - \text{proj}_{\vec{a}} \vec{b})$$



$$\boxed{\vec{b} = \underbrace{\langle 2.03, 2.03, 2.03 \rangle}_{= \vec{p}} + \underbrace{\langle -0.03, 0.01, 0.02 \rangle}_{\vec{b} - \vec{p}}}$$

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12.4 Cross Product

Text book: (90%)

3-dim:

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

$\vec{c} = \langle c_1, c_2, c_3 \rangle$

looking for a vector that is orthog to $\vec{b}, \vec{c}$

$$\vec{b} \times \vec{c} = \left\langle \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix}, -\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix}, \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \right\rangle$$

$\begin{matrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix}$ where $\begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} = \alpha\delta - \beta\gamma$