MIDTERM PRACTICE, CPSC 421/501, FALL 2017

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See the course website for info regarding the midterm.

Sample Midterm Problems

(1) Answer true or false; if false, then provide a counterexample.
 (a) If L_1 and L_2 are regular languages, then $L_1 \cap L_2$ is regular.
 (b) If L_1 and L_2 are nonregular languages, then $L_1 \cap L_2$ is nonregular.
 (c) If L is a regular language, then L^* is regular.
 (d) If L is a nonregular language, then L^* is nonregular.
 (e) If L is regular, then L is recognizable by a Turing machine.
 (f) If L is nonregular, then L is not recognizable by any Turing machine.
 (g) If L is recognized by a NFA, then it is recognized by some DFA.
 (h) If L is recognized by a NFA with n states, then it is recognized by some DFA with n states.
 (i) If $f(n) = o(2^n)$, then it is not a walk-counting function.
 (j) If $f(n) = o(2^n)$ and f has asymptotic ratio 2, then it is not a walk-counting function.
 (k) If $f(n) \sim 2^n/n$, then f is not a walk-counting function.
 (l) If $f(n) \sim 2^n$, then f is not a walk-counting function.
 (m) If $f(n) \sim (3/2)^n$, then f is not a walk-counting function.
 (n) If L is a regular language, and $f(n)$ is the number of strings of length n in L, then $f(n)$ is a walk-counting function.
 (o) If L is a nonregular language, and $f(n)$ is the number of strings of length n in L, then $f(n)$ is not a walk-counting function.
 (p) If L is a regular language, and $f(n)$ is the number of strings of length n in L, then $f(n)$ is not a walk-counting function.

Justify your answer to all questions below.

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(2) Let $\Sigma = \{0,1\}$, and let $L = \{0, 11\} \subset \Sigma^*$. Compute all possible values of

$$\mathrm{AcceptingFuture}(L,s) \stackrel{\mathrm{def}}{=} \{t \mid st \in L\}$$

as s varies over Σ^*; justify your answer. Then use these values to construct a DFA for L with a minimum number of states; explain your construction.

(3) Let $\Sigma = \{0,1\}$, and let $L = \{0, 11\} \subset \Sigma^*$. Give a Turing machine that decides L and explain how your machine works.

(4) Let $\Sigma = \{0,1\}$, and let $L = \{0^i 1^j \mid i \geq j\}$.
  (a) Give a Turing machine that decides L and explain how your machine works.
  (b) Prove that L is not regular.

(5) Let $\Sigma = \{0,1\}$, and let $L = \{1^n \mid n \text{ is a power of two}\}$.
  (a) Use the pumping lemma to show that L is not regular.
  (b) Use a fact about walk-counting functions to show that L is not regular.
  (c) Use the Myhill-Nerode theorem to show that L is not regular.

(6) Give a DFA that recognizes the language, L, of strings in $\{0,1\}^*$ such that the difference in the number of 0's and the number of ones is divisible by three. Use the procedure of obtaining a regular expression from a DFA to write a regular expression for L.

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