- 8.1 - Savitch's Thm
  8.2 - PSPACE, NPSPACE (they are equal) } Ch 8
  8.3 - We avoid messy details ← "SNEAKY"

Jump to Thm 9.20 ~~Baker~~-Gill-Solovay

Start with a Sneaky NP-complete language:

Code-Lever: SAT, 3SAT, 3COLOUR, ... NP-complete
of interest in combinatorial optimization

SNEAKY-~~NT~~M

$= \{ \langle M, w, 1^t \rangle \mid$ M is ~~de~~ non-deterministic TM that accepts input w (along one of its branches) in time t

1^t = 11...1 (length t)

(Practically uninteresting, but to prove that it is NP-complete is easy)

① SNEAKY-NTM $\in$ NP . (usually easy) (more difficult)

② $L \in NP$, $L \leq_{\text{poly time}}$ SNEAKY-NTM . (usually difficult) (fairly easy)

Prove ②: Say that $L \in NP$, recognized by a NTM, M, in time ~~n~~ n^3.

Define $f : \Sigma_L^* \to \Sigma_{\text{SNEAKY-NTM}}^*$ s.t.

- f is poly time
- $w \in L$ iff $f(w) \in$ SNEAK-NTM $\forall w \in \Sigma_L^*$

Algorithm for f:

Given w: Now write

- Description of M, w, $|||\ldots|$

  M: constant, indep of w

  w: $|w|$

  $|||\ldots|$: $|w|^3$ ← $|w|^3$ 1's

  ← LENGTH →

This f takes time roughly $\text{const} + |w| + |w|^3$

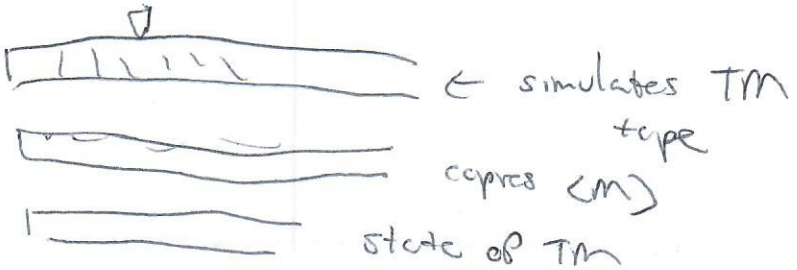
So f is poly-time (depending on knowing to run M for $|w|^3$ steps)

Prove ①: Why is SNEAKY-NTIME $\in$ NP:

$\langle M, w, \underbrace{|||\ldots|}_{t} \rangle$ given: can you build a non-det TM that negatively if w is accepted by M in t steps?

- If we are talking about non-determinism, a universal TM would work: claim: each TM step can be simulated size of $\langle M \rangle$

multi-tape



Claim: This will take $\leq O(\text{length } M + \text{Conf})$ steps per simulated step; total time $= O(t \cdot \text{length } M)$

$\leq O(|\text{input}|^2)$.

– If we are talking about non-det TM, the same thing works.
(we need to non-deterministically select the computation.

We say L is PSPACE-complete if

① $L \in$ PSPACE, and

② $L' \in$ PSPACE, then $L' \leq_{\text{poly time}} L$.

(PSPACE = poly space $= \bigcup_{k=1,2,\ldots} \text{SPACE}(n^k)$

Claim:

$$\text{SNEAKY-PSPACE} = \{ \langle M, w, 1^s \rangle \mid M \text{ is a TM that accepts } w \text{ within space } s \}$$

then SNEAKY-PSPACE is PSPACE-complete.

PSPACE