Nov 15 (1)

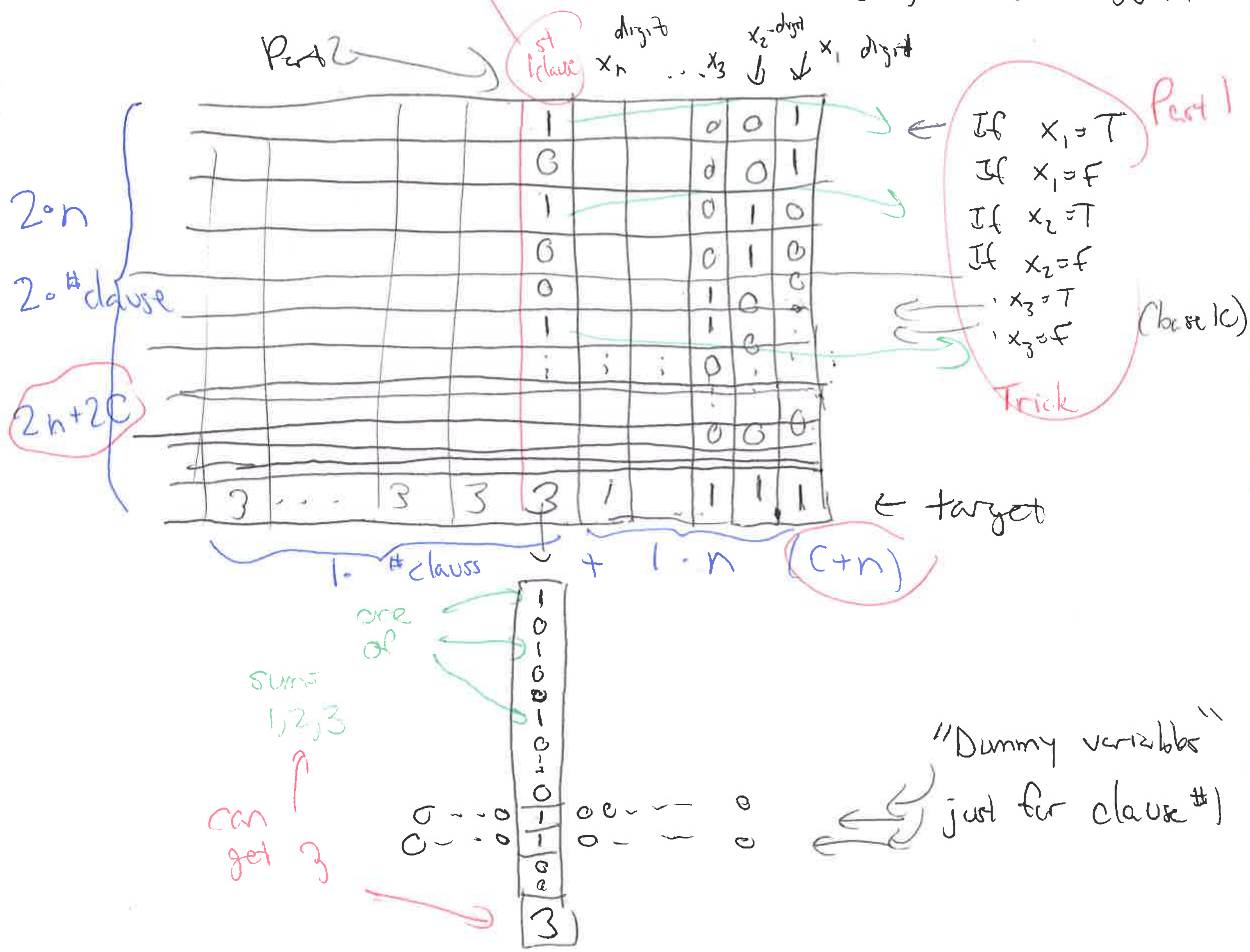
- NP-completeness:
- Oracle TM's:

Reduce 3SAT to SUBSET-SUM:

$f = (x_1 \text{ or } x_2 \text{ or } \neg x_3) \text{ and } \ldots$ ($x_1 = T$, $x_2 = T$, $x_3 = F$)

$f = f(x_1, \ldots, x_n) \in 3SAT$?

$\rightsquigarrow$ SUBSET SUM $R = R(f)$ s.t. $f \in 3SAT$ iff $R(f) \in$ SUBSET-SUM



Boolean formula n vars, c clauses

$\longrightarrow$ string of numbers of size $O((n+c)^2)$

size ~~≤~~ $O((n+c)^2) \leq O((\text{size of formula})^2)$

So reduction: R: Boolean formula $\longrightarrow$ SUBSET-SUM instance

requires size $O((\text{size formula})^2)$

So since R generates

1st number, 2nd number, ..., target

so R can be computed time $\leq O(N^3)$ poly in N, N = size formula

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So: ① SUBSET-SUM $\in$ NP (usually easy step)

usually key difficulty → ② Have reduction R reduces 3SAT to SUBSETSUM (not easy)

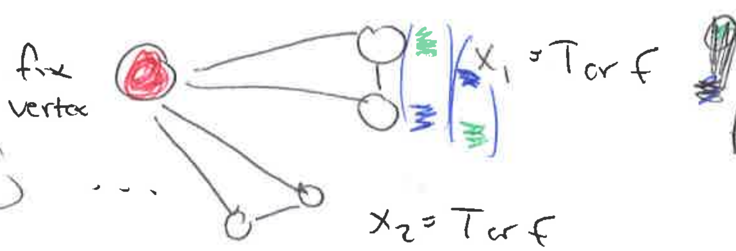
Don't forget → ③ R is poly time (usually easy)

3COLOR: Say $f \in$ 3CNF want to know if $f \in$ 3SAT

imagine $f = (x_1 \text{ or } x_2 \text{ or } \neg x_3)$ and ...

3COLOR = {⟨G⟩ | G can be 3-coloured}

$x_1 = T/F$, $x_2 = T/F$, ..., $x_n = T/F$, $C_1 = x_1 \text{ or } x_2 \text{ or } \neg x_3$, $C_2, \ldots, C_m$



Part 1

colours

Part 2 For each clause, devise a "gadget" to mean that it is satisfied

We know SUBSET-SUM is NP complete

Say

$$\text{PARTITION} = \left\{ \langle n_1, \ldots, n_k \rangle \;\middle|\; \begin{array}{l} n_1, \ldots, n_k \in \mathbb{N} \\ \text{s.t. there is} \\ I \subset \{1, \ldots, k\} \\ \text{for which} \\ \sum_{i \in I} n_i = \sum_{i \notin I} n_i \end{array} \right\}$$

e.g. $\langle 1, 2, 3, 4, 10 \rangle \in$ PARTITION

$\langle 1, 2, 3, 4, 11 \rangle \notin$ "

SUBSET-SUM problem $\{\langle n_1, \ldots, n_m, t \rangle\}$ → Partition problem that is equivalent

or

3SAT or 3COLOR

Idea: 12, 17, 37, 4, 101, target = 51 → if 1st 51, rest sum to 12+17+37+4+101 − 51 = whatever

don't touch [12, 17, 37, 4, 101, 1000051, 100 whatever]