

- NP-completeness:

- Oracle TM's:

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Reduce 3SAT to SUBSET-SUM:

$f = (x_1 \text{ or } x_2 \text{ or } \neg x_3) \text{ and } \dots$
with annotations: $x_1=T$ $x_2=T$ $x_3=F$

$f = f(x_1, \dots, x_n) \in 3SAT?$

\rightsquigarrow SUBSET SUM $R = R(f)$ s.t.

$f \in 3SAT \iff R(f) \in SUBSET-SUM$

Part 2

st clause x_n digit \dots x_3 digit \downarrow x_2 -digit \downarrow x_1 digit

$2 \cdot n$
 $2 \cdot \# \text{ clause}$

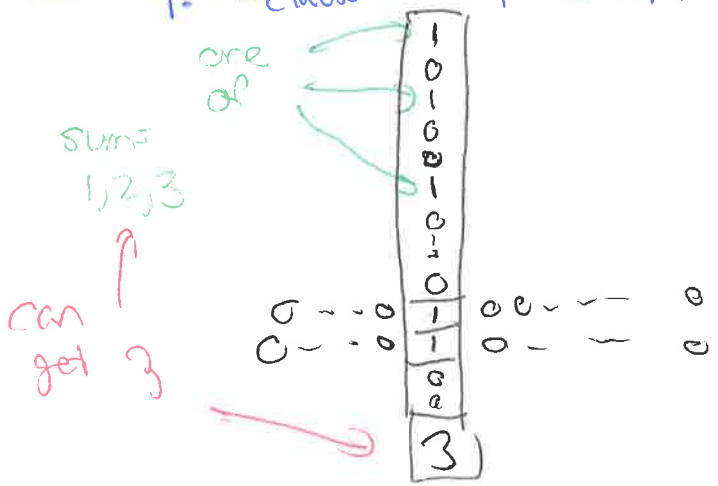
$2n + 2C$

Matrix with columns for variables and rows for clauses. Row 1: | | | | 1 | 0 | 0 | 1. Row 2: | | | | 0 | 0 | 0 | 1. Row 3: | | | | 1 | 0 | 1 | 0. Row 4: | | | | 0 | 0 | 1 | 0. Row 5: | | | | 0 | 1 | 0 | 0. Row 6: | | | | 1 | 1 | 0 | 0. Row 7: | | | | i | i | i | 0. Row 8: | | | | . | . | . | . Row 9: | | | | 0 | 0 | 0 | 0. Row 10: | 3 | . | 3 | 3 | 3 | 1 | 1 | 1 | 1

Part 1
If $x_1 = T$
If $x_1 = F$
If $x_2 = T$
If $x_2 = F$
 $x_3 = T$
 $x_3 = F$
Trick
(clause k)

target

$1 \cdot \# \text{ clause} + 1 \cdot n$ $(C+n)$



"Dummy variables" just for clause #

Boolean formula n vars, c clauses

→ string of numbers of size $O((n+c)^2)$

size ~~$O((n+c)^2)$~~ $\leq O(\text{size of formula}^2)$

So reduction: R : Boolean formula \rightarrow SUBSET-SUM instance

requires size $O(\text{size formula}^2)$

So since R generates

1st number, 2nd number, ..., target

$\&$ R can be computed time $\in O(N^3)$
 \swarrow poly in N
 $N = \text{size formula}$

So: ① SUBSET-SUM \in NP (usually easy step)

usually key difficulty
Don't forget

② Have reduction R reduces 3SAT to SUBSETSUM (not easy)

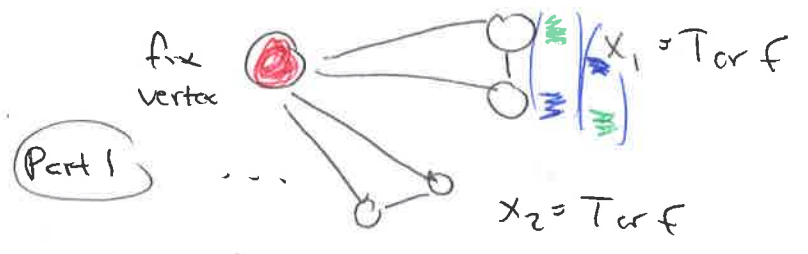
③ R is poly time (usually easy)

3COLOR: Say $f \in 3CNF$ want to know if $f \in 3SAT$

imagine $f = (x_1 \text{ or } x_2 \text{ or } \neg x_3)$ and ...

3COLOR = $\{ \langle G \rangle \mid G \text{ can be 3-colored} \}$

$x_1 = T/F, x_2 = T/F, \dots, x_n = T/F, C_1 = x_1 \text{ or } x_2 \text{ or } \neg x_3, C_2, \dots, C_m$



Part 2: For each clause, devise a "gadget" to mean that it is satisfied
 colors $\neq \neq \neq$

We know SUBSET-SUM is NP complete

Say

$$\text{PARTITION} = \left\{ \langle n_1, \dots, n_k \rangle \mid \begin{array}{l} n_1, \dots, n_k \in \mathbb{N} \\ \text{st. there is} \\ I \subseteq \{1, \dots, k\} \\ \text{for which} \\ \sum_{i \in I} n_i = \sum_{i \notin I} n_i \end{array} \right\}$$

e.g. $\langle 1, 2, 3, 4, 10 \rangle \in \text{PARTITION}$
 $\langle 1, 2, 3, 4, 11 \rangle \notin \text{PARTITION}$

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SUBSET-SUM problem

$\{ \langle n_1, \dots, n_m, t \rangle \}$

or

3SAT or 3COLOR

Partition problem that is equivalent

Idea: $\{ 12, 17, 37, 4, 101 \}$ target = 51

don't touch $\{ 12, 17, 37, 4, 101, \text{10000051}, \text{100 whatever} \}$

if LA 51
 rest sum to
 $12+17+37+4+101$
 $- 51$
 $= \text{whatever}$