Nov 6 (1)

Cook-Levin: SAT $\in$ P $\Rightarrow$ NP = P.

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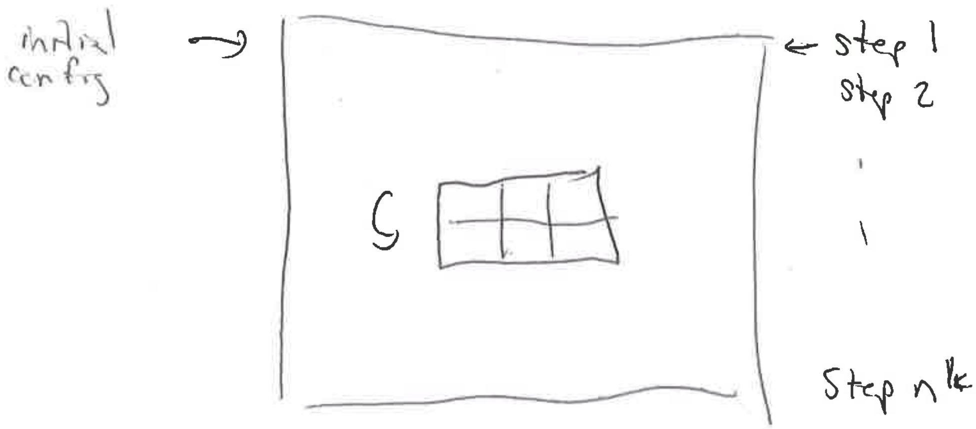
$L \in NP$, Non-det machine M recognizes L

in n^k time



$X_{i,j,\gamma} = \begin{cases} 1 & \text{if step } i \text{, cell } j \text{, we see } \gamma \in \Gamma \\ 0 & \text{otherwise} \end{cases}$

$\varphi_{\text{one thing}}$ Exactly one thing written in each cell

AND

φ_{start} What we see on step 1 is the initial conf

AND

" " " " " 2 is allowable from step 1

AND " " " " 3 " " " 2

AND " " " " 4 " " " 3

⋮

AND

" " " " n^k " " " $n^k - 1$

AND " " " " n^k is ~~an~~ ~~accepti~~ the accepting state

Boolean expressions of $X_{i,j,\gamma}$ then done

Long procedure: take input $w_1, \ldots, w_n$

spits out Boolean formula $f = f(X_{ijs}) = f(X_{ijs})(w_1, \ldots, w_n)$

size $\le$ ~~poly~~ $O(n^{2k})$, $f = f(X_{ijs})(w_1, \ldots, w_n) \in SAT \Leftrightarrow M$ accepts w

$\varphi_{\text{one thing}}$ means $\forall i, j$ $\exists$ one $s \in \{\#\} \cup Q \cup \Gamma$

s.t. x_{ijs} = True (others are false)

Say $i = j = 1$

AND AND AND AND AND AND ...

$(x_{1,1,\#}$ or $x_{1,1,q_0}$ or $x_{1,1,q_1}$ or ... or $x_{1,1,\gamma_m})$

$(\neg(x_{1,1,\#}$ and $x_{1,1,q_0}))$

$(\neg(x_{1,1,\#}$ and $x_{1,1,q_1}))$

⋮ goes on for $\binom{S'}{2}$ formulas

↖ #vars is $S' = 1 + |Q| + |\Gamma|$

write for all $i, j = 1, \ldots, n^k$

So if L recognized by M in time n^k

There's a function $\Sigma^* \xrightarrow{f} \underbrace{\text{Boolean formulas}}_{\text{strings in } \{0,\ldots,9,\, x,\, (,\,),\, \text{and},\, \neg,\, \text{XOR}, \ldots\}}$

s.t. $w \in L \iff f(w) \in SAT$.

Reducibility: This means $L \leq_p SAT$

L (is reducible)$_{\text{(poly time)}}$ SAT