- Midterms likely to come back end of next week, ~~maybe~~ possibly sooner
- Today: Cook-Levin Theorem (Sipser §7.4)

(How to make a quick $10^6 (USD)) (minus taxes)

P vs. NP:

Cook-Levin Theorem: If SAT $\in P$, then $P = NP$

i.e. "SAT is NP-complete"

Cor: If you can prove SAT $\in P$ or $\notin P \Rightarrow$ $10^6.

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P = Poly Time Languages = $\bigcup_{k=1}^{\infty} TIME(n^k)$

NP conceptual idea...

$= \bigcup_{k=1}^{\infty} NTIME(n^k)$

non-deterministic Turing machines

Deterministic TM

Non-deterministic

$NTIME(n^k)$

$n = |input|$

each branch has to step in n^k

In NP: SAT, 3COLOUR, SUBSET-SUM, PARTITION, ...

description

SAT = { ⟨f⟩ | Boolean formulas, f, that are satisfiable }

3COLOUR = { ⟨G⟩ | G has a 3-colouring } ⊂ Σ^*

some alphabet Σ "reasonable"

Boolean formula $f(x_1, \ldots x_n) = (x_1 \text{ AND } (\text{NOT } x_2)) \text{ OR } (x_3 \text{ OR } x_4)$

describe by alphabet $\Sigma = \{\wedge, \neg, \vee, (,), x, 0, \ldots 9\}$

$x_{12} \rightarrow x, 1, 2$ string, etc.

$f(x_1, \ldots x_n)$ is satisfiable if ∃ assignment $x_i \rightarrow$ TRUE/FALSE

s.t. $f(x_1, \ldots, x_n) =$ TRUE.

SAT ∈ NP { guess x_1 { TRUE / FALSE , x_2 { T / F , ... then verify }

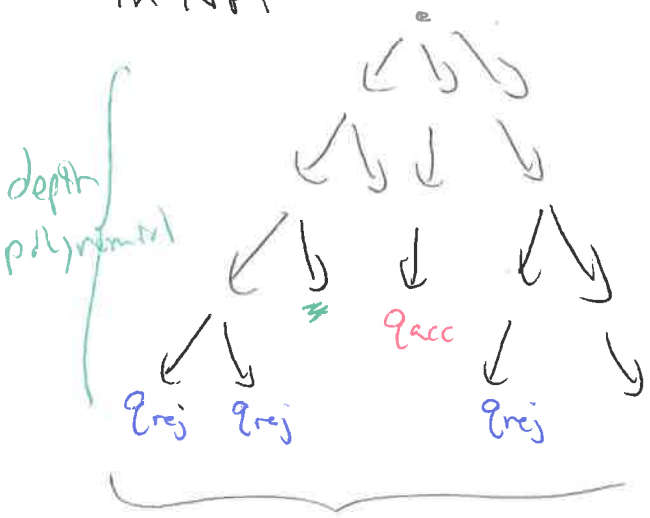
UNSAT = { ⟨f⟩ | f has no satisfying assignment } ∈ NP ???

Rem: If $L \in P$, $L^{comp} \in P$

It's not known if $L \in NP \overset{?}{\Longrightarrow} L^{comp} \in NP$???

What's Going On?

in NP:



Non-deterministic TM "accepts" if at least one path is q_{accept}

give opposite

UNSAT

=

SAT: Possible Outcomes of f

F F T T

{ T/F, T/F, T/F, T/T }

SAT = at least one T

UNSAT: all are F

=

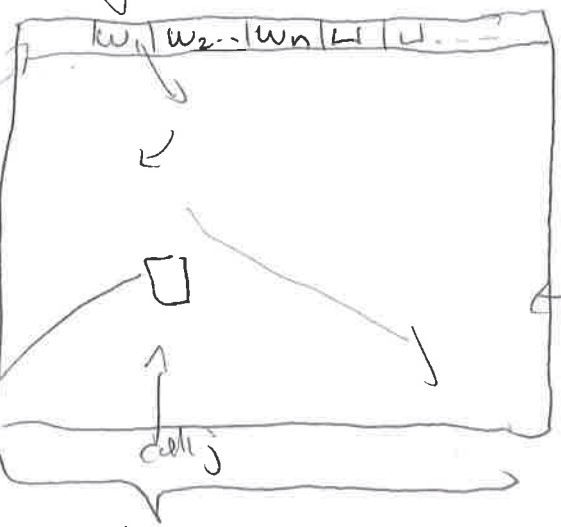
Cook-Levin Theorem: If $L \in NP$, say that M is a non-det Turing machine recognizing L in time n^k (some k). Then for each input w to M, $n = |w|$, there is a [horrible looking] Boolean formula that is

① straightforward to generate

② $f(w)$ = formula for input w on M size $O(n^{2k})$

③ w is accepted by M $\iff$ $f(w) \in$ SAT

Non-det Turing machine



config 1
poss config 2
poss config 3
⋮
n^k

1 to n^k

cell(i,j) → # → state ∈ Q
tape symbol ∈ Γ

$X_{i,j,s}$ = { true if s in cell(i,j)
false if not

s

~~At most~~ $X_{1,1,s}$ Exactly one s with T others F