MIDTERM PRACTICE, CPSC 421/501, FALL 2017

JOEL FRIEDMAN

$L = \{1^p \mid p \text{ prime}\} = \{1^2, 1^3, 1^5, 1^7, \dots\}$, $L^* = 1^2 1^*$
$\bar{L} = \{1^n \mid n \text{ perfect square}\}$ $\bar{L}^* = 1\,1^*$

See the course website for info regarding the midterm.

Sample Midterm Problems

$L_1 \cap L_1^{comp} = \emptyset$ regular

(1) Answer true or false; if false, then provide a counterexample.

T (a) If L_1 and L_2 are regular languages, then $L_1 \cap L_2$ is regular.
F (b) If L_1 and L_2 are nonregular languages, then $L_1 \cap L_2$ is nonregular. L_1 = nonreg $= \{1^p \mid p \text{ prime}\}$
T (c) If L is a regular language, then L^* is regular.
F (d) If L is a nonregular language, then L^* is nonregular. L_1 L_1^*
T (e) If L is regular, then L is recognizable by a Turing machine.
F (f) If L is nonregular, then L is not recognizable by any Turing machine. $\{0^n 1^n\}$
T (g) If L is recognized by a NFA, then it is recognized by some DFA.
F (h) If L is recognized by a NFA with n states, then it is recognized by some DFA with n states. NFA) $= \{\varepsilon\}$ DFA
F (i) If $f(n) = o(2^n)$, then it is not a walk-counting function. $f(n)=1$
T (j) If $f(n) = o(2^n)$ and f has asymptotic ratio 2, then it is not a walk-counting function.
T (k) If $f(n) \sim 2^n/n$, then f is not a walk-counting function.
F (l) If $f(n) \sim 2^n$, then f is not a walk-counting function. $f(n) = 2^n$
T (m) If $f(n) \sim (3/2)^n$, then f is not a walk-counting function.
T (n) If L is a regular language, and $f(n)$ is the number of strings of length n in L, then $f(n)$ is a walk-counting function.
F (o) If L is a nonregular language, and $f(n)$ is the number of strings of length n in L, then $f(n)$ is not a walk-counting function.
F (p) If L is a regular language, and $f(n)$ is the number of strings of length n in L, then $f(n)$ is not a walk-counting function.

$L = \{$ ~~$1^n \mid n$ is~~
$L = \{s \in \{0,1\}^* \mid$ s if binary is not a perfect square$\}$

Justify your answer to all questions below.

Research supported in part by an NSERC grant.

(2) Let $\Sigma = \{0, 1\}$, and let $L = \{0, 11\} \subset \Sigma^*$. Compute all possible values of

$$\text{AcceptingFuture}(L, s) \stackrel{\text{def}}{=} \{t \mid st \in L\}$$

as s varies over Σ^*; justify your answer. Then use these values to construct a DFA for L with a minimum number of states; explain your construction. Myhill-Nerode DFA construction

(3) Let $\Sigma = \{0, 1\}$, and let $L = \{0, 11\} \subset \Sigma^*$. Give a Turing machine that decides L and explain how your machine works. Regular & TM decidable

(4) Let $\Sigma = \{0, 1\}$, and let $L = \{0^i 1^j \mid i \geq j\}$.
 (a) Give a Turing machine that decides L and explain how your machine works.
 (b) Prove that L is not regular. Pumping Lemma / Myhill-Nerode

(5) Let $\Sigma = \{0, 1\}$, and let $L = \{1^n \mid n \text{ is a power of two}\}$.
 (a) Use the pumping lemma to show that L is not regular.
 (b) Use a fact about walk-counting functions to show that L is not regular.
 (c) Use the Myhill-Nerode theorem to show that L is not regular.

(6) Give a DFA that recognizes the language, L, of strings in $\{0, 1\}^*$ such that the difference in the number of 0's and the number of ones is divisible by three. Use the procedure of obtaining a regular expression from a DFA to write a regular expression for L. DFA → reg exp

Department of Computer Science, University of British Columbia, Vancouver, BC V6T 1Z4, CANADA, and Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, CANADA.

E-mail address: jf@cs.ubc.ca or jf@math.ubc.ca

URL: http://www.math.ubc.ca/~jf

T/F:

(k) $f(n) \sim 2^n/n$

Asy Ratio $f(n) = \underbrace{\text{AsyR}(2^n)}_{2} \cdot \underbrace{\text{Asym}(1/n)}_{1} = 2$

We knew $f(n)$ asym $\rho = 2$

But $f(n) = o(\rho^n)$ since $\underbrace{f(n)}_{2^n/n} = o(2^n)$

$L_{\text{unary primes}} = \{1^2, 1^3, 1^5, 1^7, \ldots\}$

L^* : $1^2, (1^2)^2 = 1^4, (1^2)^3 = 1^6, \ldots$ all even n, 1^n

$(1^2)1^3 = 1^5, (1^2)^2 1^3 = 1^7, (1^2)^3 1^3 = 1^9$ ... odd n, $n \geq 5$

[Even $\{1^2, 1^3\}^* = \{1^2, 1^3, 1^4, 1^5, 1^6, 1^7, \ldots\}$

$L_{\text{square}} = \{1, 1^4, 1^9, 1^{16}, 1^{25}, \ldots\}$, $L^*_{\text{square}} = \{1, 1^2, 1^3, \ldots\}$

?

(o) L non-regular, but $f(n)$ = strings in L length $\leq n$ might be a walk counting function...

$\{0^m 1^m\}$: $f(n) = \begin{cases} 1 & \text{if } n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

$\{s \in \{0,1\}^* \mid |s| = n$, and is smallest prime $\geq 2^{n-1}\}$ $f(n) = 1$

$\{0^n 1^n\} \cup \{0^m 1^{m+1}\}$ $f(n) = 1$

3 ways to show a language is not regular:

$L = \{1, 1^2, 1^4, 1^8, 1^{16}, 1^{32}, \ldots\}$

(a) Pumping Lemma: say pumping length is p:

any $s \in L$: $\underbrace{x\ y}_{\le p}\ z$ but $xz, xyz, xy^2z, \ldots \in L$

1 $|xy| \le p$ 2 $y \ne \varepsilon$ 3

length(xy^2z) = length(xyz) + length(y)

~~Take some k with $2^k >$~~

The length gap between $1^{2^k}, 1^{2^{k+1}} = 1^{2 \cdot 2^k}$ is 2^k

$L = \{ \ldots 1^{2^k}, 1^{2^{k+1}}$

nothing of length $2^k + p$ if $p < 2^k$

So choose k s.t. $2^k > p$. Take $s = 1^{2^k}$.

$s = \underbrace{1111}_{xy} \ldots 1 = 1^{2^k}$

xy length $\le p$, length $y \le p$ length$(xy^2z) \le 2^k + p$

$xy^2z = 1^{2^k + q}$, $1 \le |y| = q < 2^k$. So $xy^2z \notin L$.

$L = \{1^1, 1^2, 1^4, 1^8, 1^{16}, \ldots\}$

(b) Walk-counting:

If DFA/directed graph has $\leq p$ states/vertices, then

$$f(n) = c_1 f(n-1) + c_2 f(n-2) + \ldots + c_p f(n-p)$$

$f(n)$ = # strings in L length n :

$f(1) = 1,\ f(2) = 1,\ f(3) = 0,\ f(4) = 1, \underbrace{\ldots}_{f(n)=0 \text{ if } n \neq \text{ power of } 2}\ f(8) = 1 \quad f(16) = 1$

Choose k s.t. $2^k > p$

$$f(2^{k+1}) = c_1 \underline{f(2^{k+1}-1)} + \ldots + c_p \underline{f(2^{k+1}-p)}$$

bigger than 2^k
smaller than 2^{k+1}

0

Myhill Nerode:

$$AF(L, \varepsilon) = \{1, 1^2, 1^4, \ldots\}$$

$$AF(L, 1^2) = \{\varepsilon, 1^2, \ldots\}$$

$$AF(L, 1^4) = \{\varepsilon, 1^4, \ldots$$

$$\vdots$$

$$AF(L, 1^{2^{100}}) = \{\varepsilon, 1^{2^{100}}, \ldots\}$$

All different