

Oct 25 ①

"Acceptance Problem"

Thm: $A_{\text{TM}} = \{\langle M, w \rangle \mid M \text{ is Turing machine that accepts } w\}$ is undecidable.

Pf: Assume H is a T.m. that decides A_{TM} .
Use H to construct a new T.m., D as follows:
on input w ① check if $w = \langle M \rangle$ for some M .
② if so, D outputs the (negation) of H on $\langle M, \langle M \rangle \rangle$.
if not, doesn't matter what you do (halt: $q_{\text{acc}}, q_{\text{rej}}$)

[Opposite: accept if H ends in q_{rej} , reject -- q_{acc}]

Given D : What is the result of D on input $\langle D \rangle$?

Either { D accept $\langle D \rangle$: $\Rightarrow H$ rejects $\langle D, \langle D \rangle \rangle$ by def of H, A_{TM} }
 D rejects $\langle D \rangle$: -- accept .. $\Rightarrow D$ accepts $\langle D \rangle$

Imagine that we use U , universal TM, for H

U given $\langle M, w \rangle$ ^{"interprets"} M on w { q_{acc} if M on w accepts
 q_{rej} -- M on w "rejects"

Why is there no contradiction?

D is built on running H (now U) and doing opposite

If $\langle M, w \rangle$ accepts, then D on $\langle M, w \rangle$ q_{rej}

$\langle M, w \rangle$ is not accepted { either rejects -- loops -- } D loops

Can D_{fake} accept $\langle D_{\text{fake}} \rangle$ \Rightarrow contradiction

"reject" \Rightarrow "
 D_{fake} must loop!!

D_{fake} doesn't halt

Theorem: Take any universal TM, U . Let ②

D' be built as follows: on input s

① check if $s = \langle M \rangle$ of some M

② if so, "run" U on $\langle M, \langle M \rangle \rangle$
if not, do whatever

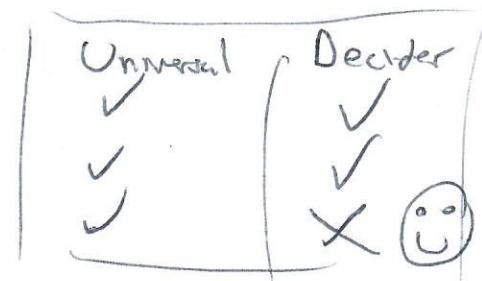
③ if/when we halt, print the opposite $\begin{array}{l} \text{acc} \rightarrow \text{rej} \\ \text{rej} \rightarrow \text{acc} \end{array}$

Then: D' on input $\langle \text{D}' \rangle$, doesn't halt, i.e. loops.

Feeding $\langle \text{D}', \langle \text{D}' \rangle \rangle$ into D' (really will run U on it
then do something at the end)

M accept w : we halt in q_{acc}

M does not reject w : $\begin{cases} \dots \dots q_{\text{rej}} \\ \text{we loop (don't halt)} \end{cases}$



Theorem: Complement of $\overline{A_{\text{TM}}}$, $\widetilde{A_{\text{TM}}}$,

$\widetilde{A_{\text{TM}}} = \{ s \mid \begin{array}{l} s \text{ is not equal to } \langle M, w \rangle \\ \text{s.t. } M \text{ accepts } w \end{array} \}$ is not recognizable

(Analogy: If L_1 and L_2 regular, $L_1 \cap L_2$ regular.)

So if L_1, L_2 not regular, one of L_1, L_2 is not regular)

Show L_1 is not regular, it's enough to show that $L_1 \cap ab^*c^*$ is not regular)

Fact: If L is recognized by a TM and $\overline{L} = \{ \dots \dots \dots \dots \dots \}$ then L is decidable)

Since A_{TM} is recog, $\widetilde{A_{\text{TM}}}$ isn't, otherwise A_{TM} would be decidable