"Acceptance Problem"

Thm: $A_{TM} = \{ \langle M, w \rangle \mid$ M is Turing machine that accepts w $\}$ is undecidable.

Pf: Assume H is a T.m. that decides A_{TM}. {always halt in q_{acc} or q_{rej}

SELF CONTAINED

Use H to construct a new T.m., D as follows:

— on input w ① check if $w = \langle M \rangle$ for some M.

② if so, D outputs the $\boxed{\text{opposite}}$ (negation) of H on $\langle M, \langle M \rangle \rangle$.

if not, doesn't matter what you do (halt: q_{acc}, q_{rej})

[Opposite: accept if H ends in q_{rej}, reject .. q_{acc}]

Given D: What is the result of D on input $\langle D \rangle$?

Either { D accept $\langle D \rangle$: $\Rightarrow$ H rejects $\langle D, \langle D \rangle \rangle \xrightarrow{\text{by def of } H, A_{TM}}$ D, on input $\langle D \rangle$ rejects

D rejects $\langle D \rangle$: .. accept .. $\Longrightarrow$ D accepts $\langle D \rangle$

Imagine that we use U, universal TM, for H

U given $\langle M, w \rangle$ "interprets" "simulates" M on w { q_{acc} if M on w accepts; q_{rej} — — rejects; loops M on w "loops"

Why is there no contradiction?

D^{fake} is built on running H (now U) and doing opposite ~~exit~~ fake

If $\langle M, w \rangle$ accepts, then D^{fake} on $\langle M, w \rangle$ q_{rej}

$\langle M, w \rangle$ ~~is~~ not accepted { either rejects .. q_{acc}; loops .. D loops $\leftrightarrow$ D^{fake} doesn't halt

Can D^{fake} accept $\langle D^{fake} \rangle$ $\Rightarrow$ contradiction

" reject " $\Rightarrow$ "

D^{fake} must loop !!

<u>Theorem</u>: Take any universal TM, U. Let D' be built as follows: on input s

1. check is $s = \langle M \rangle$ of some M
2. if so, "run" U on $\langle M, \langle M \rangle \rangle$
   if not, do whatever
3. if/when we halt, print the opposite acc → rej, rej → acc

Then: D' on input $\langle D' \rangle$ doesn't halt, i.e. loops.

Feeding $\langle D', \langle D' \rangle \rangle$ into D' (really will run U on it then do something at the end

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M accept w : we halt in q_{acc}

M does not ~~reject~~ w : { " " " q_{rej}
{ we loop (don't halt)

Universal	Decider
✓	✓
✓	✓
✓	✗

<u>Theorem</u>: Compliment of A_{TM}, $\overline{A_{TM}}$,

$\overline{A_{TM}} = \{ s \mid s$ is not equal to $\langle M, w \rangle$ s.t. M accepts $w \}$ is not recognizable

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(Analogy: If L_1 and L_2 regular, $L_1 \cap L_2$ regular.
So if $L_1 \cap L_2$ not regular, one of L_1, L_2 is not regular)

Show L_1 is not regular, its enough to show that $L_1 \cap ab^*c^*$ is not regular)

(Fact: If L is recognized by a TM and $\overline{L}$ " " " " " } then L is decidable.)

Since A_{TM} is recog, $\overline{A_{TM}}$ isn't, otherwise A_{TM} would be decidable