Oct 11 (1r)

3 types of regularity:

(1) DFA (§1.1) (2) NFA (§1.2) (3) Reg Exp (§1.3)

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3 tests for regularity + min # of states

(1) Asymptotic ratio tests (2) Pumping Lemma

(3) Myhill-Nerode Thm

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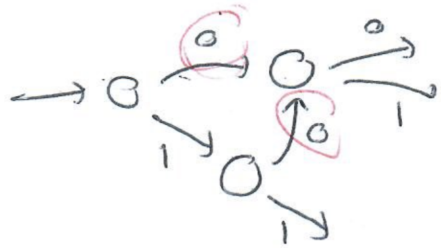
Question: Say $L$ is regular. For a string $s \in \Sigma^*$, let $s^{\text{Reverse}} = \sigma_n \sigma_{n-1} \cdots \sigma_1$ where $s = \sigma_1 \cdots \sigma_n$, $\sigma_i \in \Sigma$. Let

$$L^{\text{Reverse}} = \{ s^{\text{Reverse}} \mid s \in L \}.$$

Is $L^{\text{Reverse}}$ regular ??

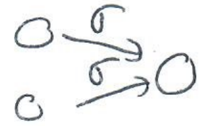
- If $M$ is a DFA for $L$, is there a totally obvious way to turn $M$ (very simply) into a DFA recognizes $L^{\text{Reverse}}$?

e.g.



etc.

DFA $\leadsto$ NFA



reverse arrows get NFA

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§1.3 Reg Exp:

$1^*\circ(0,110)^*\circ 112\circ(2,11)\circ(121,0)\circ 2^*$ describes $L$

$L^{\text{Reverse}}$ $\quad (R_1 \circ R_2)^{\text{rev}} = R_2^{\text{rev}} \circ R_1^{\text{rev}}$, $(R_1 \cup R_2)^{\text{rev}} = R_1^{\text{rev}} \cup R_2^{\text{rev}}$

$(R_1^*)^{rev} = (R_1^{rev})^*$.

Regular Exp: Built from $\circ, \cup, *$ + basic $\emptyset$, $\varepsilon$, $\sigma \in \Sigma$.

1.4: → Asymptotic Ratio Tests

- Pumping Lemma

→ Myhill-Nerode Thm ← Tells you when 2 states can be merged into one

Derived situations:

$L_2 = \{0^n 1^n 0^m \mid n, m \in \mathbb{Z}_{\geq 0}\}$

say we know that $\boxed{L_1 = \{0^n 1^n \mid n \in \mathbb{Z}_{\geq 0}\} \text{ is \underline{not} regular}}$

but...

for sake of contradiction → If $L_2$ regular, $\{0^* 1^*\}$ is regular

$\Rightarrow$ $\underbrace{L_2 \cap \{0^* 1^*\}}$ would be regular

$\underbrace{\{0^m\} \cup \{0^n 1^n\}}$ = ~~$L_1$, we know $L_1$ is not regular...~~

then $\cap$ $(0,1)^* 0 1 (0,1)^*$ $\{0^n 1^n \mid n \geq 1\} = (01)^+$

$\Rightarrow$ $\{0^n 1^n \mid n \geq 1\} \cup \{\varepsilon\}$ regular $= \{0^n 1^n \mid n \geq 0\}$

$(\{0^m\} \cup \{0^n 1^n\}) \cap (0,1)^* 1 (0,1)^*$

Say $L = \{ 0^n 1^m 2^t \mid n = m+t \quad n,m,t \in \mathbb{Z}_{\geq 0} \}$

is this reguler?

RegExp for $L$ (assuming it exists, to get contradiction)

$(0,1)^* 2^* (0, 11, 201)^*$ - -

$0 \to a$

$1 \to b$ transforms $L$ to $\{a^n b^m b^t \mid n = m+t\}$

$2 \to b$ $= \{a^n b^n\}$ which is not reguler

Fact: If $L$ is reguler $\xrightarrow{\text{substitution}}$ get reguler expression

$\to$ reguler

$L$ is reguler then any Substitution($L$) is reguler

So if $L$ is a language, and some Substitution($L$) is not reguler

then $L$ is not reguler

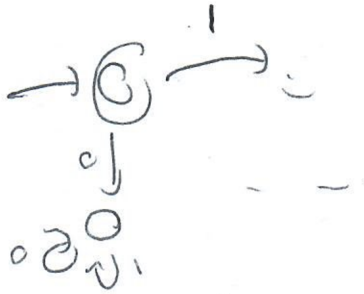
Not so useful, but most akin to reductions in NP-completeness

Myhill-Nerode

$L = \{ s \in \{0,1\}^* \mid$ each 0 is isolated, i.e. each 0 is followed by and preceded by a 1 $\}$

$= (101, 1)^*$

How small a DFA?



s vary over all $\{0,1\}^*$

Myhill-Nerode : AcceptingFutures$(L, s)$

AcceptingFutures$(L, \varepsilon) = L = (101, 1)^*$ ← 2

A. F $(L, 1) = \{ 01L \cup L \}$ ← 4

- - - $(L, 0) = \emptyset$ ← 1

- - - $(L, 0\text{ any}) = \emptyset$

- - - $(L, 10) = \{ 1L \}$ ← 3

- - $(L, 11) =$

110

111

Build the DFA - - -