When is L non-regular? (§1.4)

- $\frac{1}{2}$ way there — Standard test: Pumping Lemma
- ✓ — Asymptotic ratio test: More direct (when it work); uses facts on linear algebra not covered in CPSC 421
- next, after — Myhill-Nerode Thm (Exercises 1.51, 1.52 of [Sip]) More difficult, but tells you the minimal number of states in DFA recognizes, and tells you how to build it

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<u>Pumping Lemma</u>: Say that L is regular and accepted by a DFA of p states or fewer. Then if $s \in L$ and $|s| \ge p$ we can write $s = xyz$ s.t.

① $xz, xyz, xy^2z, xy^3z, \ldots \in L$ ② $y \neq \varepsilon$, i.e. $|y| \ge 1$

③ $|xy| \le p$.

Many variants: ... $s \notin L$ ... $xy^iz \notin L$ $i = 0, 1, \ldots$

Claim: $L = \{0^n 1^n \mid n = 0, 1, \ldots\}$ is not regular.

<u>Proof</u>: Say L is regular and accepted by a DFA of p states. $\boxed{\text{Now consider } s = 0^p 1^p \in L.}$ Then $s = xyz$ such that ① – ③ above hold. Since $s = 0^p 1^p = xyz$ and $|xy| \le p$ we have $x = 0^a, y = 0^b, z = 0^{p-a-b} 1^p$.

Chosen to get contradiction

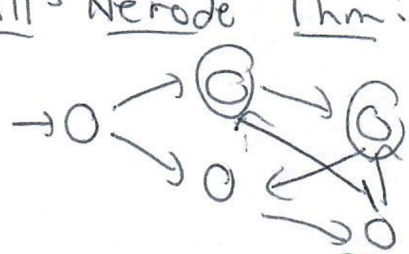
$[\, s = 0^p 1^p = \underbrace{0 \ldots 0}_{p\ 0\text{'s}} 1 \ldots 1 = xyz$ so x, y all 0's $]$

So $xz, xyz, xy^2z, xy^3z, \dots, xy^iz, \dots \in L$

$x = 0^a$, $y = 0^b$ From 2 $y \neq \varepsilon$, so $b \geq 1$, and

$0^a z \in L$, $0^{a+b}z$, $0^{a+2b}z, \dots \in L$, so $0^{a+bi}0^{p-a-b}1^p$ is in L. This is impossible, since $xz = 0^a 0^{p-a-b} 1^p = 0^{p-b}1^p$ cannot be in L.

Myhill - Nerode Thm:



etc

Here's when $x, y \in \Sigma^*$ can't land in the same state of a DFA recognizing L.

Say that for some $w \in \Sigma^*$: $x w$, $y w$ } one is in L the other isn't, then x, y are in different states

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Define for $L \subset \Sigma^*$, $x \in \Sigma^*$

$$\text{AcceptingFuture}(L, x) = \{w \in \Sigma^* \mid xw \in L\}$$

$L_1 = \{0^n 1^m \mid \text{any } n, m\} = 0^*1^*$ regular

$L_2 = \{0^n 1^m \mid n = m\} = \{0^n 1^n \mid n = 0, 1, \dots\}$ (non-regular).

What are futures: $\text{AcceptingFutures}(L_1, x) = ?$

$\text{AccFuture}(L_1, \varepsilon) = \{w \in L_1\} = L_1 = 0^*1^*$

" $(", 1) = \{w \mid 1w \in L_1\} = 1^*$

" $(L_1, 10) = \{w \mid 10w \in L_1\} = \emptyset$

" $(L_1, 0) = \{w \mid 0w \in L_1\} = 0^*1^*$

" $(L_1, 00011) =$ - - - $= 1^*$

⋮

How many of these are there?

$L_2 = \{0^n 1^n\}$:

$\text{AccFuture}(L_2, 000) = \{w \mid 000w \in L_2\} =$

$\{1^3, 0^1 1^4, \ldots, 0^a 1^{a+3}, \ldots\}$

$\text{AccFuture}(L_2, 00) = \{1^2, 01^3, 0^2 1^4, \ldots\}$

$\text{AccFuture}(L_3, 0^4) = \{1^4, 01^5, \ldots\}$

$\text{AccFuture}(L_3, 0^r) = \{1^r, 01^{r+1}, \ldots\}$

How many possible futures?

Ans: Infinite

Construction of DFA for L_1