Oct 4 1

§1.4 Nonregular languages

(§1.1 DFA, §1.2. NFA, §1.3 Reg Exp)

§1.4: - Given a language → ① Is it regular, is there a DFA recognizing the language

There are may variants.. → ② If so, how many states does the DFA need?

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1.4. Focus: - Pumping Lemma, variants

- Myhill Nerode theorem (Exercises in [Sip])

Most direct test → - Walk Counting Functions (Asymptotic Ratios)

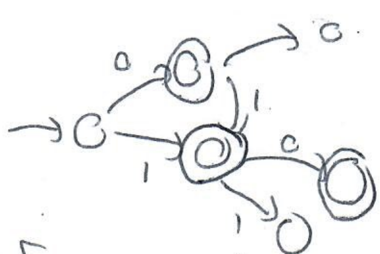
Example:

$$L_{3/2\text{-length}} = \left\{ s \in \{0,1\}^* \;\middle|\; \begin{array}{l} s \text{ in binary is a} \\ \text{number } \leq (3/2)^{|s|} \end{array} \right\}$$

$|s|$ = length of s, i.e. $|s| = n$, s in binary, is it $\leq (3/2)^n$.

Claim: $L_{3/2\text{-length}}$ is not regular (use asymptotic ratio test)

DFA:



etc.

If L is regular, and

$f(n)$ = # strings of length n in L

$f(n)$ counts # walks length n starting in $\{q_{init}\}$ ending F

$D = (Q, \Sigma, \delta, q_{init}, F)$

\# strings length n in $L_{3/2\text{-length}}$ =
\# of strings in binary, length n $\leq (3/2)^n$

size $\{0, 1, 2, \ldots \lfloor (3/2)^n \rfloor\}$ $= (3/2)^n + O(1)$

floor?

So asyptotic ratio of $f(n) = (3/2)^n + O(1)$

Same as " " " " $(3/2)^n$

so this ratio is $3/2$.

By one of ~~test~~ for walk-counting functions, $f(n)$ is not a walk-counting function. $\Rightarrow$ $L_{3/2\text{-length}}$ is not regular.

Recall: If $f(n)$ is ~~walk counting and~~ has asymptotic ratio ρ then $f(n)$ is not walk counting if

(1) ρ is rational but not an integer

(2) $f(n) = o(\rho^n)$

If $f(n)$ is walk-counting, on a directed graph with $\leq p$ vertices, then there are integers $c_1, \ldots, c_p$ s.t. f satisfies

states

$$f(n) = f(n-1)c_1 + f(n-2)c_2 + \cdots + f(n-p)c_p.$$

Example: $L_{\text{prime}} = \{ s \in \{0,1\}^* \mid s \text{ in binary is a prime number} \}$.

(Quick 10^6 Riemann hypothesis..)

$\pi(x)$ = # primes between 1 and x, $\pi(x) \sim \dfrac{x}{\log_e x}$. (Prime # Theorem)

$f(n) =$ # strings length n in L_{prime}

$$f(n) \sim \frac{2^n}{\log_e(2^n)} = \frac{2^n}{n \log_e(2)}$$

(Asymp ratio $f(n) = 2^n \cdot \frac{1}{n} \cdot \frac{1}{\text{const}}$) is 2

But $f(n) \sim \frac{2^n}{n \cdot \text{cost}} = o(2^n)$. Hence L_{primes} is not regular

Method 2: Pumping Lemma:

$$L = \{ s \in \{0,1\}^* \mid s = 0^m 1^m \text{ for some } m \}$$

$$= \{ \varepsilon, 01, 0011, 000111, \ldots \}$$

Claim: L is not regular

Idea: Say that L is recognized by a DFA with 100 states

etc.

Look at $s = 0^{100} 1^{100} \in L$

Consider what state you're in after

two of these wound in same state $q \in Q$

$$\left\{ \begin{array}{l} \varepsilon \longleftrightarrow q_{init} \\ 0 \longrightarrow \text{same state} \\ 00 \\ \vdots \\ \underbrace{0 \ldots 0}_{100} \longrightarrow \end{array} \right\}$$

101 strings
≤ 100 states

Maybe $0^3, 0^{87}$ wind up in same state

$\rightarrow q$ — reach on 0^3
— reach on $0^{87} = 0^3 0^{84}$

If so

$0^3 \to q \xrightarrow{\text{another } 0^{84}} q \xrightarrow{\text{another } 0^{84}} q \to \ldots$

$0^3 \, 0^{84} \cdots 0^{84} \to$ state q

$0^{100} 1^{100} \in L$ I know $0^{87} \to q$ then $0^{13} 1^{100} \to$ some accepting state

So $0^3 (0^{84})^m \, 0^{13} 1^{100}$ is accepted

So $0^3 \, 0^{84000} \, 0^{13} 1^{100}$ is accepted

Pumping Lemma: If L is regular, accepted by a machine of at most p states, and s is any string in L of length at least p, then there are $xyz = s$ s.t.

(1) $y \neq \varepsilon$ (2) $|xy| \leq p$ (3) $xy^m z \in L$ for all m.