1.1 Regular Languages, DFAs ← last time

1.2. NFA's, etc. non-determinism ← today

1.3 Regular Expressions

1.4 Non-regular languages ← (proofs)

finite state machines

$L_3 = \{ 1^m \mid m \text{ is divisible by } 3 \}$

=

→ Non-regular language: $\{ 1^m \mid m \text{ is a perfect square} \}$

$\{ 1^m \mid m \text{ is a prime} \}$

$\{ 0^m 1^m \mid m \in \mathbb{N} \}$

($0^m = \underbrace{0 \cdots 0}_{m}$, $1^m = \underbrace{1 \cdots 1}_{m}$, $0^m 1^m$ or $0^m \circ 1^m = (\underbrace{0,\ldots,0}_{m}, \underbrace{1,\ldots,1}_{m})$

↑ concatenate

=

If L, L' are two languages over Σ:

1.1 ⇒ $L \cup L'$ = union of sets: $\{ s \in \Sigma^* \mid s \in L$ ~~or~~ $s \in L' \}$

1.1 → $L \cap L'$ = intersection: $\{ s \in \Sigma^* \mid s \in L \text{ and } s \in L' \}$

1.1 → $L \circ L'$ = concatenation of L and L' $= \{ s = xy \mid x \in L, y \in L' \}$

- L^* $= \{ s = x_1 \cdots x_k$ some k, s.t. $x_1, \ldots, x_k \in L \}$

e.g. $L = \{000, 010\}$, $L^* = \{\varepsilon, 000, 010, 000010, 000000, \ldots\}$

Eg. $L_3 = \{1^m \mid m \text{ is divis by } 3\}$

$L_5 = \{1^m \mid m \text{ is divis by } 5\}$

$L_3 \cup L_5 = \{1^m \mid m \text{ is divis by } 3 \text{ OR } m \text{ is divis by } 5\}$

$L_3 \cap L_5 = \{1^m \mid m \text{ is divis by } 3 \text{ AND } m \text{ is divis by } 5\}$

$= \{1^m \mid m \text{ divis by } 15\}$

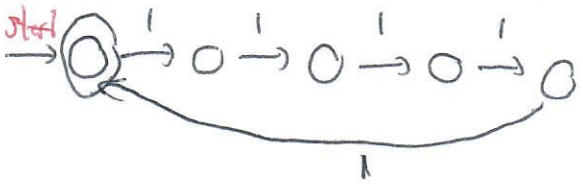
$L_3 L_5 = \{s = xy \mid x \in L_3, y \in L_5\}$

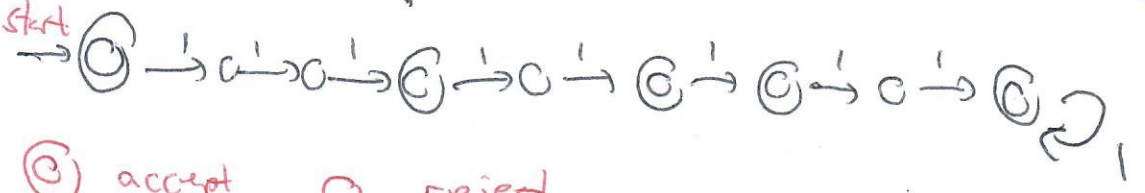
$= \{1^m \mid m = 3a + 5b,\ a, b \in \mathbb{Z}_{\geq 0}\}$

$= \{\varepsilon, 1^3, 1^5, 1^6, 1^8, 1^9, \ldots\}$ ← Machine

→ Thm: (Proven in 1.2): If L, L' regular, so is $L \circ L'$.

L_3: 

L_5: 



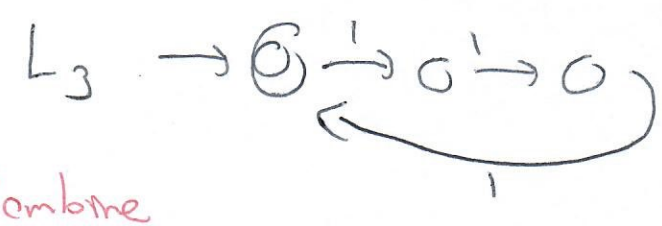
(3) accept ○ reject

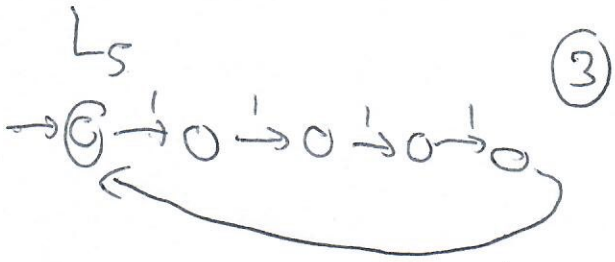
=

Non-deterministic FA: not deterministic (1) state → state 1, state 2, state 3

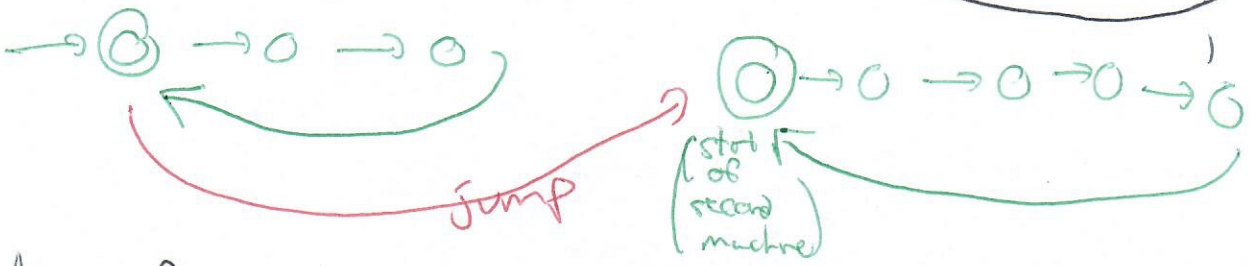
(2) state — no arrows out of it i.e. 0 choices

(3) state → 0, 1 → state; 1 → jump → state





Combine



Flexibility of non-determinism:

Define: NFA (non-deterministic FA) to be $M=(Q,\Sigma,\delta,q_{init},F)$

where blah blah blah (as before) but

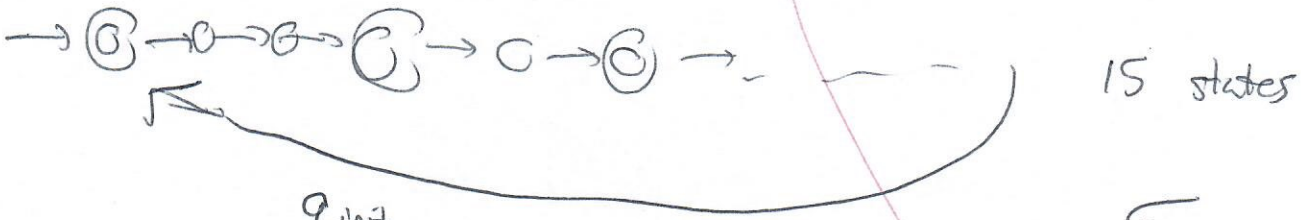
$$\delta: Q \times \Sigma_\varepsilon \to \text{Power}(Q) \quad , \Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$$

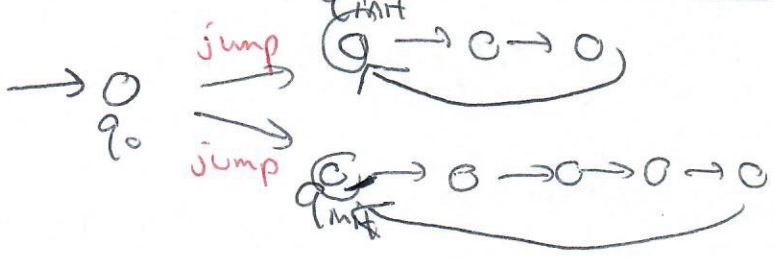
- Q: at a state
- Σ_ε: read symbol
- $\text{Power}(Q)$: some collection of possible states
- ε: jump

=

L_3, L_5: $L_3 \cup L_5 = \{1^m \mid m \text{ divides } 3 \text{ OR } m \text{ divides } 5\}$

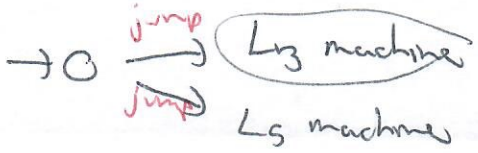
$= \{1^m \mid m \bmod 15 = 0, 3, 5, 6, 9, 10, 12\}$





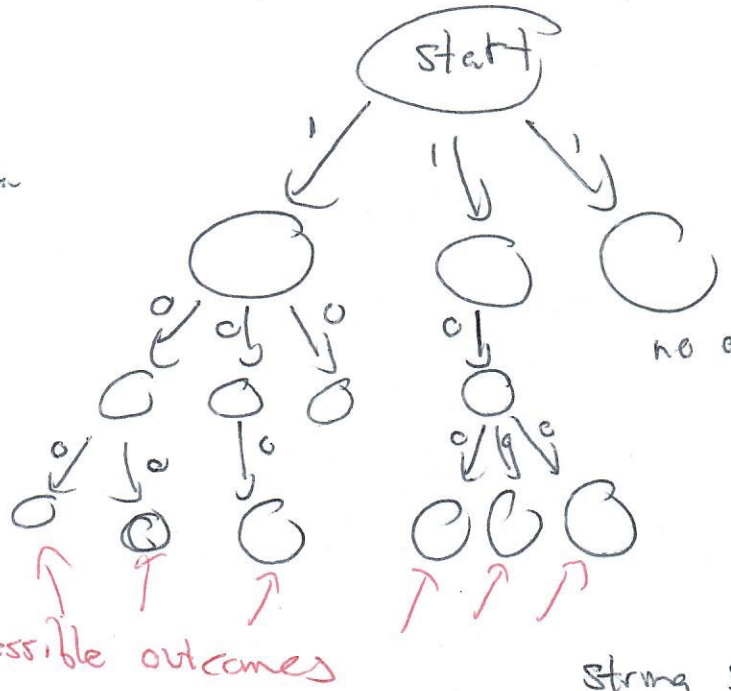
saves some effort

$\delta(q_0, \varepsilon) = \{q_{init}, q_{init}\}$



Idea of non-determinism:

String s = 100...



possible braches

no options for 0 step

String s is accepted by the NFA if at least one possible branch is accepted