§1.1 Regular Languages, DFAs

§1.2. NFAs

Idea: Alphabet Σ (symbols), Have a "long string", element of Σ^*, want a simple notion of an algorithm

Formalism: A deterministic finite automaton is a 5 tuple $M = (Q, \Sigma, \delta, q_0, F)$ where

Q = set of "states", Σ = alphabet, $\delta : Q \times \Sigma \to Q$

q_0 = "start" or "initial" state, F = accepting or final states.

Each machine M gives rise to the language $L(M)$ of strings over Σ^* that it accepts in the following way...

$\Sigma = \{0,1\}$, let ENDS_IN_0 $= \{ s \in \{0,1\}^* \mid s \text{ ends in } 0 \}$

$[\, S = 11101011..01 \,]$

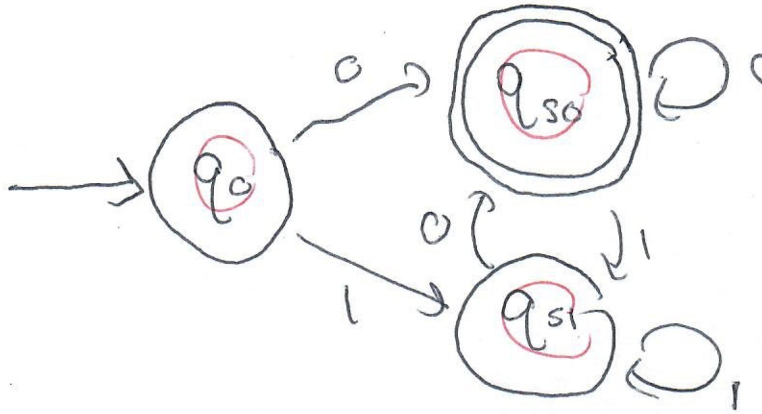
We "accept" s if it ends in

---

simpler (?)

we could combine and get a machine with fewer states

Let's introduce some simplifying notation



q_0 = start

q_{s0} = just saw 0

q_{s1} = " " 1

names needed?

Formally:

$Q = \{q_0, q_{s0}, q_{s1}\}$

$\Sigma = \{0, 1\}$

$\delta : Q \times \Sigma \to Q$

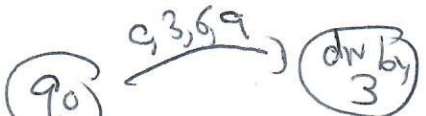
So... $\delta(q_0, 0) = q_{s0}$, $\delta(q_0, 1) = q_{s1}$, $\delta(q_{s0}, 0) = q_{s0}$, $\delta(q_{s0}, 1) = q_{s1}$

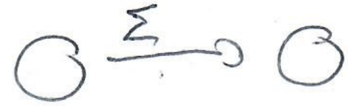
$\delta(q_{s1}, 0) = q_{s0}$, $\delta(q_{s1}, 1) = q_{s1}$. Start q_0. Accepting $F = \{q_{s0}\}$

Formalism given $s = \sigma_1 \sigma_2 \sigma_3 \dots \sigma_n$ we start at initial state, q_0

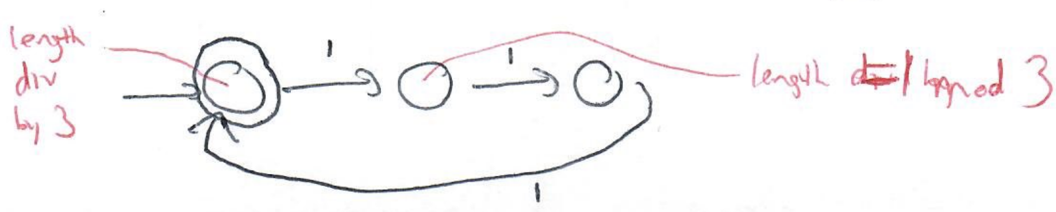
based on σ_1 we transition to $\delta(q_0, \sigma_1) = q_1$, $\delta(q_1, \sigma_2) = q_2, \dots$

we "accept s" if $q_n = \delta(q_{n-1}, \sigma_n) \in F$.

If 



Let $\Sigma = \{1\}$, let $L_3 = \{1^m = \underbrace{11\dots1}_{m}$ s.t. m is divisible by 3$\}$



$L_5 = \{ 1^m \mid m \text{ is divisible by } 5 \}$

We say a language is <u>regular</u> is there is DFA that accepts that language.

Fact: If L is regular, and L' is regular (over same Σ) then $LL' = \{ s \mid s = s_1 s_2 \text{ with } s_1 \in L,\ s_2 \in L' \}$ is also regular.

? easy

Example: What is $L_3 L_5$ with L_3, L_5 as above?

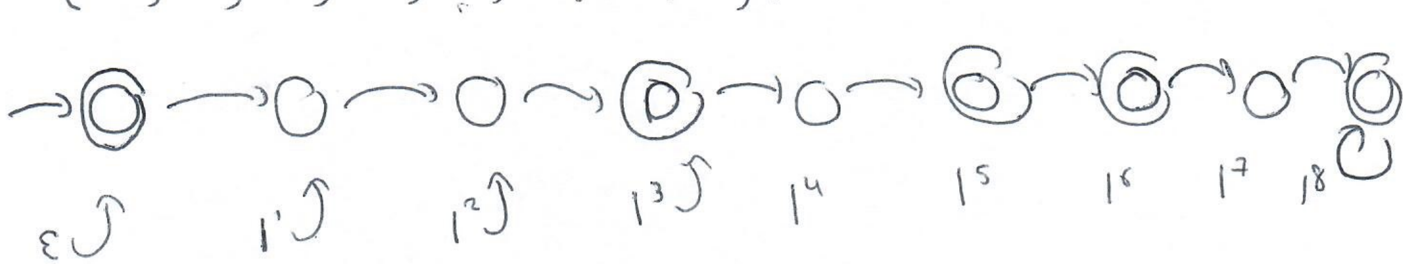
$L_3 L_5 = \{ 1^a 1^b \mid a \text{ divis by } 3,\ b \text{ divis by } 5 \}$

1^a a divis by 3 $\quad 1^b$ b divis by 5

Fact: ... $L \cap L'$ is regular, $L \cup L'$ is regular, L^*

$L_3 L_5 = \{ 1^n \mid n = a+b,\ a \text{ divis } 3,\ b \text{ by } 5 \}$

$= \{ \varepsilon, 1^3, 1^5, 1^6, 1^8, 1^9, 1^{10}, 1^{11}, \ldots \}$



What is minimum # of states?