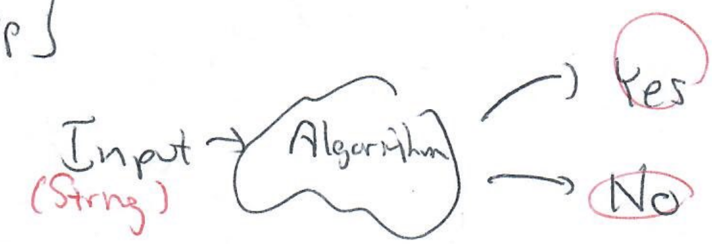
- Decision Problems ⟷ Languages
- Algorithms ⟷ Machine/Program ⟷ String
- Thm: Some decision problems can't be solved.

Today: Start regular languages, DFA's, Chapter 1

Ch 1, 2, 3, -- most [Sip]

Decision problem:



DIVISIBLE_BY_3 = $\{ S \in \{0,1,\ldots,9\}^* \mid$ S, as base 10 integer, is divisible by 3 $\}$

PRIMES = $\{ S \in \{0,\ldots,9\}^* \mid$ S, as base 10 integer, is a prime $\}$

We fix an alphabet A, here $\{0,\ldots,9\}$, a decision problem over A is just a language over A, i.e. element of Power(A^*).

Use ALL CAPS for languages.

Often use $A = \{0,1\}$, $\{a,b\}$, ...

PALINDROME = $\{ s \in \{a,b\}^* \mid s$ is its own reverse $\}$

= $\{ \varepsilon, a, b, aa, bb, abba, abbaabba, \ldots \}$

$s = s_1 \ldots s_n$, its reverse $s_n s_{n-1} \ldots s_1$

$(ab)^{\text{reverse}} = (ab)^R = ba$

DIVISIBLE_BY_3, PRIMES $\longleftrightarrow$ Languages over $\{0, \ldots, 9\}$

$\longleftrightarrow$ element Power$(\{0, \ldots, 9\}^*)$

Algorithm: Chapter 1: Finite automaton, variants
Chapter 2: Pushdown automaton
" 3: Turing machine
C++ program
Java program

(interpret)

Remark: C++ program $\in$ ASCII* (interpret)
Java " $\in$ ASCII* (")

Chapter 1: Finite automaton:

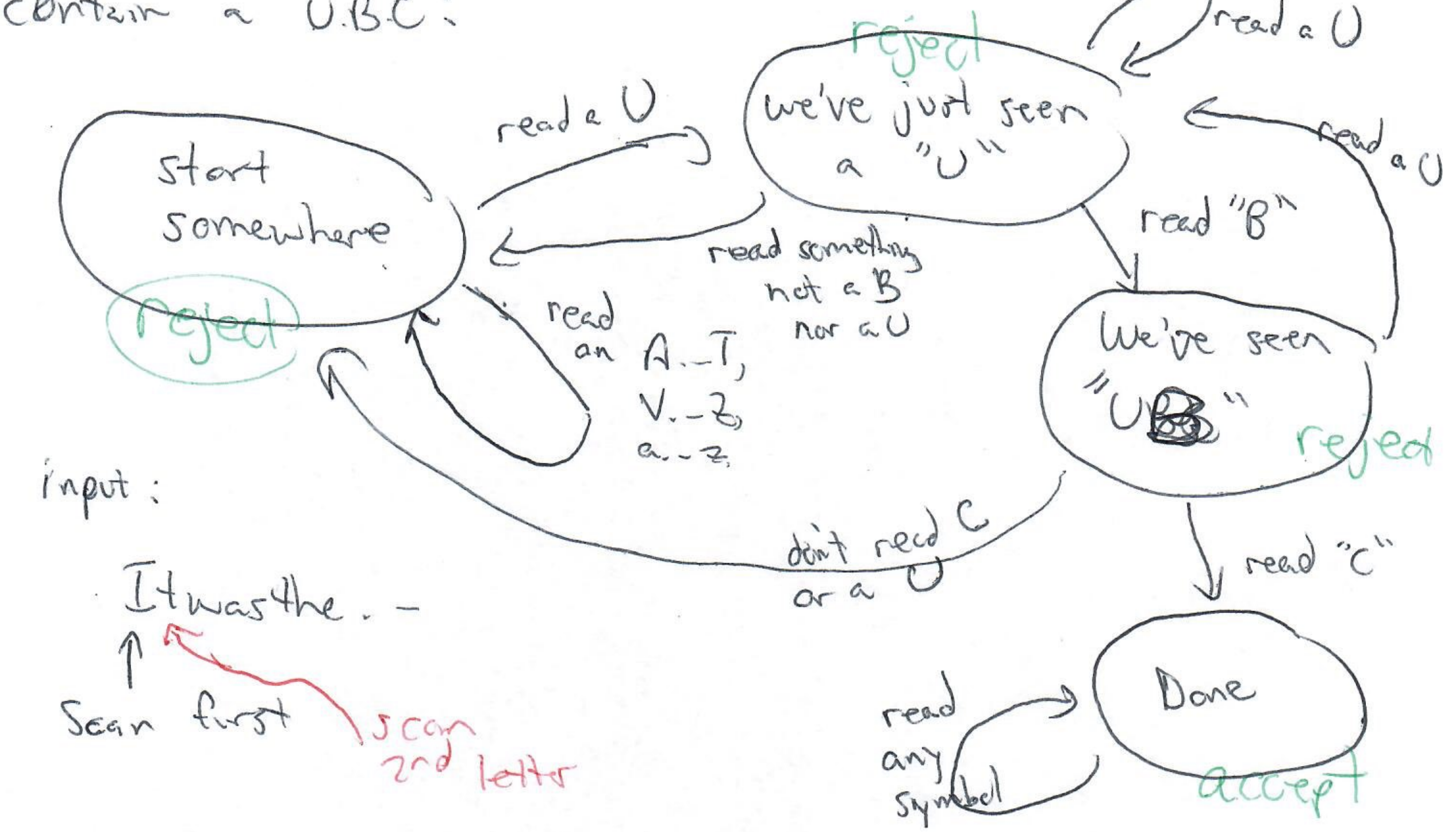
Input 318921549 6 : is it divisible by 3 ?

Input Itwasthebestoftimesitwastheworst ... THEEND :
does it contain a "UBC" in it?

Finite automaton: scan the input: first letter/symbol, second letter, ..., switches from state to state ..

Does string = "Itwasthebest... THE END."

contain a U.B.C:



Description of a DFA:

states: a set Q, q_0 = initial state

have some states "accepting states" ; F

We need rule: $\delta : Q \times \Sigma \longrightarrow Q$, Σ = alphabet

← transition of states

(start, U) ⟼ (just seen "U")

(start, A) ⟼ (start)

(start, B) ⟼ (start)

⋮

Formality: A deterministic finite automaton over alphabet Σ, has state set Q, initial state q_0, ...

Rough idea:

Algorithm: list of states : { start somewhere, just seen 0, -- }

Rule δ: $Q \times \Sigma \to Q$ (finite amount of info)

Accepting states initial state (finite amount of info)

Really begin DFA's, what problems they can, can't solve (Chapter 1)

Alphabet, say ASCII = { a,…, z, A…Z, 0,…, 9, ·, ; … }

Any DFA ∈ ASCII*

any C-program ∈ ASCII*

A problem, decision problem, language, element of Power(ASCII*)

(PRIMES, DIV_BY_3, HAS_A_UBC_IN_IT)

So {DFAs}, {Turing Machines}, {C-program}, —

countably infinite, bijection ℕ

{Languages over any alphabet} is uncountable

Therefore, regardless of model!

ASCII* ⟶ The problem is solves ({ as T.M., as DFA, as C-program })

⟶ Not be a valid algorithm

Since there is no surjection: ASCII* ⟶ Power(ASCII*)

there are languages not solved by any algorithm.

Why?

Thm: There is no surjection $S \to Power(S)$ (for any S).

Proof: If there is a surjection $f: S \to Power(S)$ let

$$T = \{ s \in S \mid s \notin f(s) \}.$$

If f is a surjection, there is a $t \in S$ s.t. $f(t) = T$

.... contradictions.