- O, o for $\mathbb{N} \to \mathbb{R}$
- O, o in [Sip] for functions $\mathbb{N} \to \mathbb{R}_{\geq 0}$
- If $f, g : \mathbb{N} \to \mathbb{R}$ usually write
  - $f(n) = o(g(n))$ if $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$
  - $f(n) = O(g(n))$ if there are n_0, C s.t.

$$|f(n)| \leq C\,|g(n)| \text{ for } n \geq n_0$$

So $n - 2 = n + O(1)$

$$n^2 + 3n = n^2 + \underbrace{O(-n)}_{\text{unusual, usually } O(n)}$$

Main tools:

$$\underbrace{3n^2 - 2n + (\log n)^{24}}_{3n^2 + o(3n^2)} = O(\text{ mess })$$

- Second article: Self-Referencing, Paradoxes, Etc.
  - Real points: - Get used to counting, set theory, --
    - Get ☺ result

☺ result: There are a lot of problems that can't be solved. !!!!!

Countable / Solutions ⟷ Programs: some type of string/word

Uncountable Problems ⟷ Decision problems: some type of language

§1-4 Article on Self-referencing,

2nd ☺ The halting problem can't be solved. !!!!

- Some paradoxes, some use, §1-2
- ~~Pidgeon hole~~ principle -- -- ⟷ finite form:
  drawer

You have 40,000 students at UBC, have at least 2 with same last 5 digits of --

-- 650,000 people in Vancouver, -- -- -- 6 digits of --

There are ∞ versions --

- Self-reference
  - This sentence is a lie. Q: Is the sentence T/F?
    self-refrence — not true. — negation
  - Let S be the set of all sets that don't (neg) contain themselves. [Russell's paradox]

Does S contain itself?

Claim: $S \in S \Rightarrow$ S does not contain itself ☹

$S \notin S \Rightarrow$ S does contain S ☹

So (1) don't care (2) probably can be fixed, my stuff, $\mathbb{N}$, $\mathbb{R}$, ... won't be affected

(3) rethink things, new theorems, ...

The physio heals only ~~the~~ shoulders of all people (in town) who don't (neg) heal their own shoulders.

Q: If physio injures her shoulder, someone heals her shoulder?

This sentence refers to the smallest positive integer not described by a sentence in English of fewer than twenty-two words. !!!

Say 117,235,729,981

Theorem: Let S be a set. Then $\text{Power}(S) = 2^S$ is larger than S.

Recall: $\text{Power}(S) = 2^S$ is the set of all subsets of S.

$$\text{Power}(\{1,2,3\}) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$$

T is larger than S — for T, S finite sets — if $|T|$ = size of T $> |S|$.

Say S, T (even if they are infinite) have the same size if there is a bijection between them, i.e. $f: S \to T$ s.t.

f is (1) injection, (2) surjection

$f: S \to T$ injection if $f(s_1) = f(s_2) \Rightarrow s_1 = s_2$

$$\begin{aligned} 1 &\mapsto a \\ 2 &\mapsto b \\ 3 &\mapsto a \end{aligned}$$

not an injection $\{1,2,3\} \to \{a,b\}$

e.g. $1 \mapsto a$
$2 \mapsto c$ is injection $\{1,2,3\} \to \{a,b,c\}$
$3 \mapsto b$

" " $\{1,2,3\} \to \{a,b,\dots,z\}$

=

$f: S \to T$ is a surjection:

If $t \in T$, then some $s \in S$ has $f(s) = t$.

$1 \mapsto a$
$2 \mapsto d$ not surjection $\{1,2,3\} \to \{a,b,\dots,z\}$
$3 \mapsto e$

---

map $\mathbb{N} \to \{0,1\}$, $n \mapsto n \bmod 2$

$1 \mapsto 1$
$2 \mapsto 0$
$3 \mapsto 1$ is a surjection
$4 \mapsto 0$
$\vdots$

=

$\mathbb{N} = \{1,2,3,4,\dots\}$, $\mathbb{Z}_{\geq 0} = \{0,1,2,3,\dots\}$

$n \mapsto n-1$ is bijection, 1-1, $\begin{cases} \text{injection} \\ \text{surjection} \end{cases}$