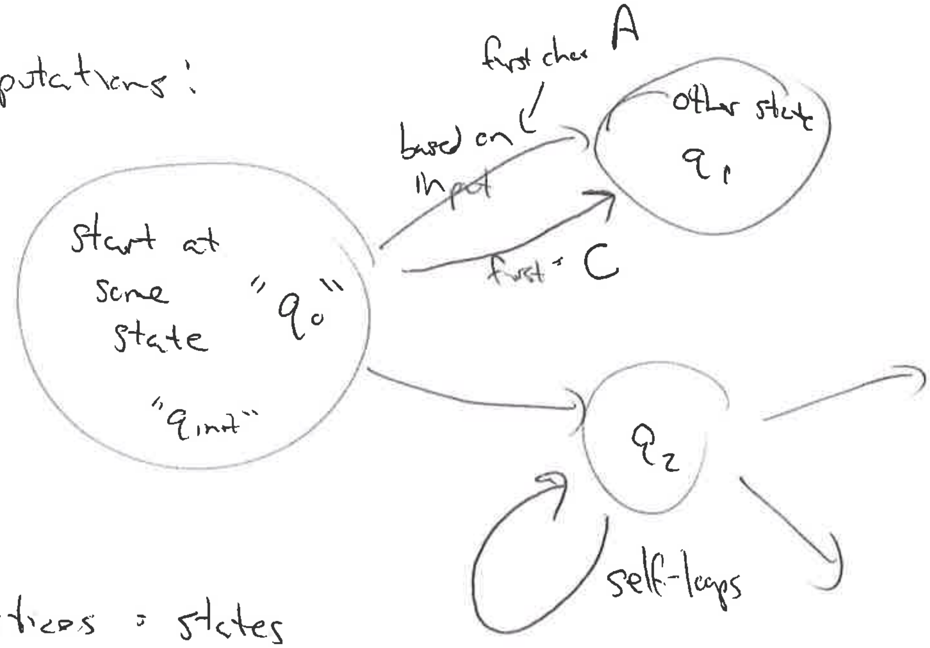
- Gradescope.com accounts via ugrad.cs.ubc.ca
- Office hours TA's — generally M, Tu
  HW due — Thursday 11:59pm
  My office — Thursday 2-3pm Math Building 210
- Piazza page exists

[- I'll make sure that you can resubmit]

Last time:

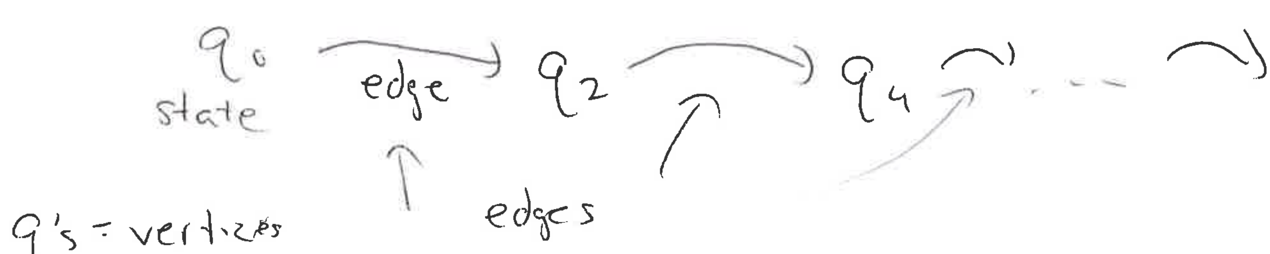
- Alphabets, strings, languages,
- Computations:



◎ vertices = states

$\longrightarrow$ edges directed (based on input to algorithm)
(or what tape heads, etc.)

Any given computation:



Formalize: Directed graph: (V, E, h, t)

V = set (finite) vertizes
E = set (finite) edges
$h, t : E \to V$

multiple edges

A <u>walk</u> in digraph $G = (V, E, h, t)$ is a sequences

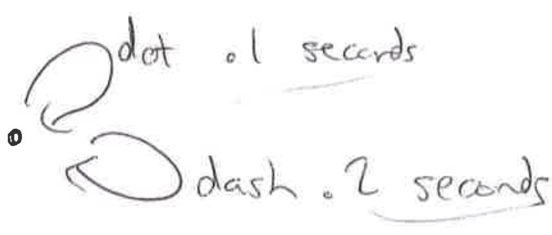
$$(v_0, e_1, v_1, e_2, v_2, \ldots, e_n, v_n)$$ $n+1$ vert, n edges

$v_0, \ldots, v_n \in V$, $e_1, \ldots, e_n \in E$ and

$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_2 \to \cdots$ $\quad h(e_i) = v_i$, $t(e_i) = v_{i-1}$ for $i = 1, \ldots, n$

"walk on <u>length</u> n". "Walk of length 0: (v_0)

"Information" 100 BPS: in one second 2^{100} possible messages



other stuff

$A = \{ \cdot, - \}$

string/words

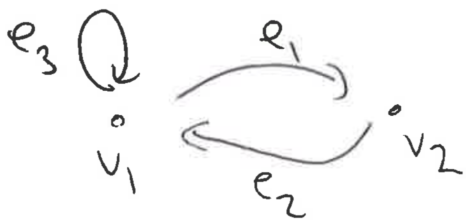
In 20 second, minute, ... how messages can you send

.1 =	dot	1 message
.2 =	dot dot, dash	2 " s
.3 =	dot dot dot, dash dot, dot dash	3 " s

Walks v_1 to v_1

dot, dot, da- -sh, da-sh, dot

7 edges $\longleftrightarrow$ .7 seconds



$e_3 =$ dot, $e_1 =$ da-, $e_2 =$ -sh

$f(n) =$ # walks length n from v_1 to v_1

dot dot da - sh dot dot
da - sh dot da - sh dot
~~dash~~ dot dot da - sh

count

$f(6) = f(5) + f(4)$

end on dot

$n \geq 2, 3$ generally $f(n) = f(n-1) + f(n-2)$

$f(1), f(2), f(3), f(4), f(5), \ldots$

$1, 2, 3, 5, 8, 13$

$f(n) =$ Fibonacci $(n+1)$

$=$ horrible expression ..

$$\zeta^2 = \zeta + 1$$

$$\left(\zeta_+^{n+1} - \zeta_-^{n+1}\right) / \sqrt{5}$$

$$\zeta = \frac{1 \pm \sqrt{5}}{2}$$

$\zeta_+ = \frac{\sqrt{5}+1}{2}$ Golden ratio (>1)

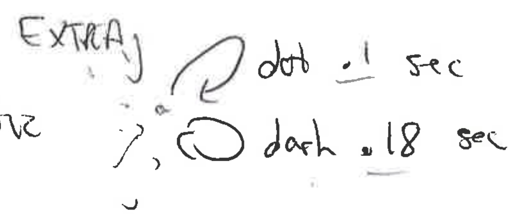
$\zeta_- = -\left(\frac{\sqrt{5}-1}{2}\right)$ this is <1, >-1

$$f(n) = \zeta_+^{n+1} \frac{1}{\sqrt{5}} + O(1) \sim \zeta_+^{n+1} \frac{1}{\sqrt{5}}$$

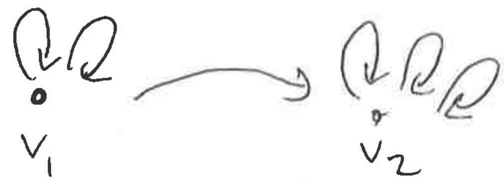
messages in 20 seconds $f(200) = \zeta_+^{201} \frac{1}{\sqrt{5}}$

-- -- k seconds $\zeta_+^{10k+1} \frac{1}{\sqrt{5}} = \zeta_+^{10k + O(1)}$

Realistic

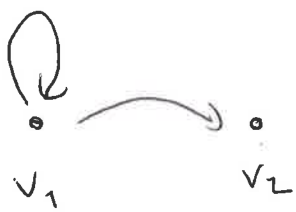


What do walk-counting functions look like?



walks from v_1 to v_2
length n

$$f(n) : \begin{cases} 1 & n=1 \\ 0 & \text{otherwise} \end{cases}$$



walks length n from v_1 ~~to~~ v_1 = 1

" " " v_1 to v_2 $= \begin{cases} 0 & n=0 \\ 1 & n \geq 1 \end{cases}$

walks length n from v_1 to v_2

$$= \begin{cases} 1 & n \text{ is odd} \\ 0 & n \text{ is even} \end{cases}$$

n edges
n steps