

## HOMEWORK #6, CPSC 421/501, FALL 2017

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Please note:

- (1) Proofs should be written out formally.
- (2) Homework that is difficult to read may not be graded.
- (3) You may work together on homework, **you must write up your own solutions individually**. You must acknowledge with whom you worked (specify their `ugrad.cs.ubc.ca` email addresses). You must also acknowledge any sources you have used beyond the textbook and two articles on the class website.
- (4) When you submit your homework to `gradescope.com`, you need to put the solutions to different problems on different pages; `gradescope.com` will ask you to identify which pages correspond to which problems. Please use the problem numbers below.
- (5) Bonus questions count for marks above the 10% homework grade.

### Homework Problems

- (1) Problem 1.42 of [Sip]. [This is also Problem 1.42 in the 2nd Edition of [Sip].]
- (2) Consider the language  $F$  of Problem 1.54 of [Sip]. [This is also Problem 1.54 in the 2nd Edition of [Sip].] Use the Myhill-Nerode theorem (see class notes) to show that  $F$  is not regular. In other words, show that

$$\text{AcceptingFuture}(F, s) \stackrel{\text{def}}{=} \{t \mid st \in F\}$$

has infinitely many possible values as  $s$  varies over all strings in  $\{a, b, c\}^*$ .

- (3) Consider the language  $F$  of Problem 1.54 of [Sip]. Setting  $L$  to be the language described by the regular expression  $ab^*c^*$ , show that  $F \cap L$  is not a regular language, using the pumping lemma. Explain why this implies that  $F$  is not regular.

- (4) Parts (b,c) of Problem 1.54 of [Sip].
- (5) Let  $L$  be given by the regular expression  $(1, 10, 1001)^*$ .
- (a) Recall that for any language  $L$  and string  $s$ ,  $\text{AcceptingFuture}(L, s)$  is defined to be
- $$\{t \mid st \in L\}.$$
- Determine the values of  $\text{AcceptingFuture}(L, s)$  for the values  $s = \epsilon$ ,  $s = 0$ ,  $s = 1$ .
- (b) Based on part (a), build the part of a DFA for  $L$  that gives the initial state,  $q_0$ , of the DFA, the transitions out of  $q_0$  upon reading either a 0 or a 1. Explicitly describe which state(s) are associated to which values of  $\text{AcceptingFuture}(L, s)$  for the values of  $s$  in part (a).
- (c) Complete part (b) to the complete description of a DFA for  $L$  by computing other values of  $\text{AcceptingFuture}(L, s)$  and associating to each distinct value a distinct state of the DFA.

**Bonus question:** 1.45 [same in 2nd edition].

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