HOMEWORK #5, CPSC 421/501, FALL 2017

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Please note:

(1) Proofs should be written out formally.
(2) Homework that is difficult to read may not be graded.
(3) You may work together on homework, **you must write up your own solutions individually.** You must acknowledge with whom you worked (specify their `ugrad.cs.ubc.ca` email addresses). You must also acknowledge any sources you have used beyond the textbook and two articles on the class website.
(4) When you submit your homework to `gradescope.com`, you need to put the solutions to different problems on different pages; `gradescope.com` will ask you to identify which pages correspond to which problems. Please use the problem numbers below.
(5) Bonus questions count for marks above the 10% homework grade.

(1) Problem 1.16 of [Sip], part (a).
(2) Problem 1.21 of [Sip], part (b).
(3) Recall that if $f\colon \mathbb{N} \to \mathbb{Z}$ is a walk-counting function, and if f has asymptotic ratio ρ, then $f(n)$ cannot be $o(\rho^n)$.
  (a) Use Stirling's formula $n! \sim \sqrt{2\pi n}(n/e)^n$ to prove that for $\binom{2n}{n} \sim 2^{2n}\gamma/\sqrt{n}$ for a constant γ. What is γ?
  (b) Use the previous part to prove that $\binom{2n+1}{n} \sim 2^{2n+1}\gamma/\sqrt{n}$ (for the same constant γ). [Hint: Consider the expression $\binom{2n+1}{n}/\binom{2n}{n}$.]
  (c) Prove that the language

$$\bigl\{ s \in \{0,1\}^* \bigm| s \text{ has } n \text{ zeros and } m \text{ ones, and } m = n \text{ or } m = n+1 \bigr\}$$

  is not regular.
(4) Recall that if $f\colon \mathbb{N} \to \mathbb{Z}$ is a walk-counting function on a graph with p vertices, then f satisfies a recurrence equation

$$f(n) = c_1 f(n-1) + \cdots + c_p f(n-p)$$

for all $n \geq p+1$, where the $c_1, \ldots, c_p$ are constants (and integers). Using this fact

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(a) prove that the language

$$\{1^m \mid 7 \text{ divides } m\}$$

is not recognized by a DFA with fewer than 7 states; and

(b) prove that the language

$$\{1^m \mid m \text{ is a perfect square}\}$$

is not regular.

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