

## HOMEWORK #10, CPSC 421/501, FALL 2017

JOEL FRIEDMAN

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Please note:

- (1) **We may only mark a subset of the problems below, depending on time constraints; the solution set we provide will solve all of the problems below.**
- (2) Proofs should be written out formally. **Your solutions should be explained: e.g., if we ask for a DFA, you should explain how it works, not merely produce a diagram of the DFA.**
- (3) Homework that is difficult to read may not be graded.
- (4) You may work together on homework, **you must write up your own solutions individually.** You must acknowledge with whom you worked (specify their `ugrad.cs.ubc.ca` email addresses). You must also acknowledge any sources you have used beyond the textbook and two articles on the class website.
- (5) When you submit your homework to `gradescope.com`, you need to put the solutions to different problems on different pages; `gradescope.com` will ask you to identify which pages correspond to which problems. Please use the problem numbers below.
- (6) Bonus questions count for marks above the 10% homework grade.

### Homework Problems

- (1) Exercise 8.4 of [Sip].
- (2) Exercise 8.6 of [Sip].
- (3) **This problem is optional, worth 0 points.**  
Give a Turing machine that takes as input,  $x \in \{0, 1\}^*$ , and (1) accepts  $x$  if  $x$  contains exactly twice as many 0's as 1's, and (2) rejects  $x$  otherwise. You must **explain how your machine works**, and **explicitly write** your choice of  $Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}}$ . To describe  $\delta$ , you may (1) list its values, or (2) use a diagram as used in Sipser's textbook (and class).

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Your solution should have the following parts:

- (a) a high-level description of the algorithm,
- (b) an explanation of what each state in  $Q$  represents in terms of your algorithm,
- (c) a list of  $\Sigma, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}$ , and
- (d) a description of  $\delta$ , either by (1) a list of its values, or (2) a diagram.

(4) **This problem is optional, worth 0 points.**

Let

$$5\text{COLOR} = \{\langle G \rangle \mid G \text{ is colorable with 5 colors}\}.$$

Show that 5COLOR is NP-complete; you may use the fact that 3COLOR is NP-complete (see Problem 7.29 of [Sip]).

DEPARTMENT OF COMPUTER SCIENCE, UNIVERSITY OF BRITISH COLUMBIA, VANCOUVER, BC V6T 1Z4, CANADA, AND DEPARTMENT OF MATHEMATICS, UNIVERSITY OF BRITISH COLUMBIA, VANCOUVER, BC V6T 1Z2, CANADA.

*E-mail address:* [jf@cs.ubc.ca](mailto:jf@cs.ubc.ca) or [jf@math.ubc.ca](mailto:jf@math.ubc.ca)

*URL:* <http://www.math.ubc.ca/~jf>