Marks

[10] **1.** Describe a Turing machine that takes as input, $x \in \{a, b\}^*$, and (1) accepts x if $|x|$ is even, and (2) rejects x if $|x|$ is odd. You should **explicitly write** and **explain** each of $Q, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}, \delta$.

Answer: For example, we may scan to the right, alternating between two states q_0 (the initial state) and q_1, and enter the appropriate accepting or rejecting state when we encounter a blank. So we may take

$$Q = \{q_0, q_1, q_{\text{accept}}, q_{\text{reject}}\}, \quad \Gamma = \{a, b, \beta\},$$

where β is the blank symbol, and set

$$\delta(q_0, x) = (q_1, x, R), \quad \delta(q_1, x) = (q_0, x, R), \quad \textbf{for } x = a \textbf{ or } x = b\textbf{, and}$$

$$\delta(q_0, \beta) = (q_{\text{accept}}, \beta, R), \quad \delta(q_1, \beta) = (q_{\text{reject}}, \beta, R),$$

with the values of δ on the accepting and rejecting states being irrelevant. (Also everything we write to the tape is irrelevant.)

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[10] **2.** Let 4SAT be the language of 4cnf's (conjunctions of disjunctions of 4 literals). Give a direct polynomial time reduction to show that 3SAT $\le_{\mathrm{P}}$ 4SAT.

Answer: A clause $y_1 \wedge y_2 \wedge y_3$ is equivalent to the redundant clause $y_1 \wedge y_2 \wedge y_3 \wedge y_3$, and performing this redundancy operation to each clause of a 3cnf yields (in polynomial time) an equivalent 4cnf. This gives the desired reduction

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[10] **3.** Let L_{agree} be (as in class) the language of $\langle M, N\rangle$ such that M and N are Turing machines that give the same result (accept, reject, or loops) on all inputs. Show that $L_{\text{yes}} \leq L_{\text{agree}}$.
[Note: L_{yes} is the language of encodings of pairs M, x where M accepts x.]

Answer: Given a pair P, x, let M be a Turing machine that (1) erases its input, (2) writes x on the tape, and (3) runs P (either by simulation or just by incorporating P into M). Let N be a Turning machine that accepts all its inputs. Then P accepts x iff M and N agree on all inputs. This gives the desired reduction from pairs P, x to pairs M, N where the former is in L_{yes} iff the latter is in L_{agree}. Hence this is a reduction of L_{yes} to L_{agree}.

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[10] **4.** Recall how we showed L_{yes} is undecidable. Assume to the contrary that there is a program, P, that decides L_{yes}. Let D be a program such that for all programs, Q,

$$\text{Result}(D, \text{EncodeProg}(Q)) = \neg\text{Result}(P, \text{EncodeBoth}(Q, \text{EncodeProg}(Q)))$$

Argue that considering the value of $\text{Result}(D, \text{EncodeProg}(D))$ leads to a contradition.

Answer: Since P is a decider, so is D, and hence D can never loop. Assume that

$$\text{Result}(D, \text{EncodeProg}(D)) = \texttt{no};$$

then

$$\neg\text{Result}(P, \text{EncodeBoth}(D, \text{EncodeProg}(D))) = \texttt{no},$$

so

$$\text{Result}(P, \text{EncodeBoth}(D, \text{EncodeProg}(D))) = \texttt{yes},$$

so

$$\text{Result}(D, \text{EncodeProg}(D)) = \texttt{yes},$$

which is a contradiction. Similarly, if we assume that $\text{Result}(D, \text{EncodeProg}(D)) = \texttt{yes}$**, then we conclude** $\text{Result}(D, \text{EncodeProg}(D))$ **is either** `no` **or** `loop`**, again a contradiction.**

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The End

Be sure that this examination has 6 pages including this cover

The University of British Columbia

Midterm Examinations - November 2009

Computer Science 421/501

Closed book examination

Time: 50 minutes

Name ______________________________ **Signature** ______________________________

Student Number____________________ **Instructor's Name** ____________________

Section Number ____________________

Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam. This exam is two-sided!

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1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

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(b) Speaking or communicating with other candidates.

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3. Smoking is not permitted during examinations.

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2		10
3		10
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Total		40