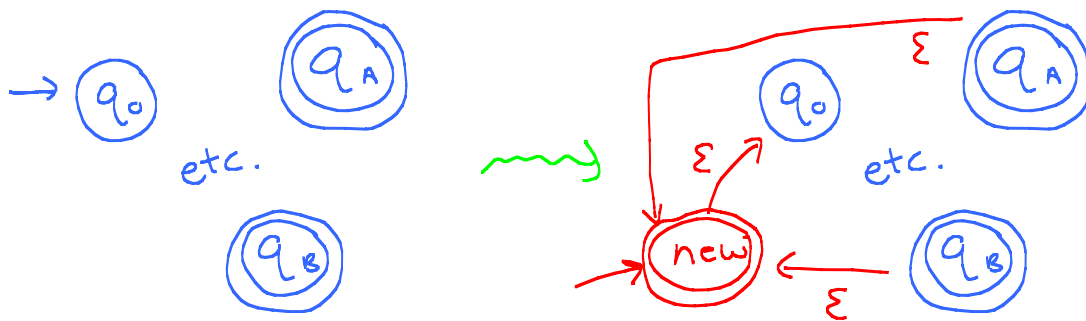
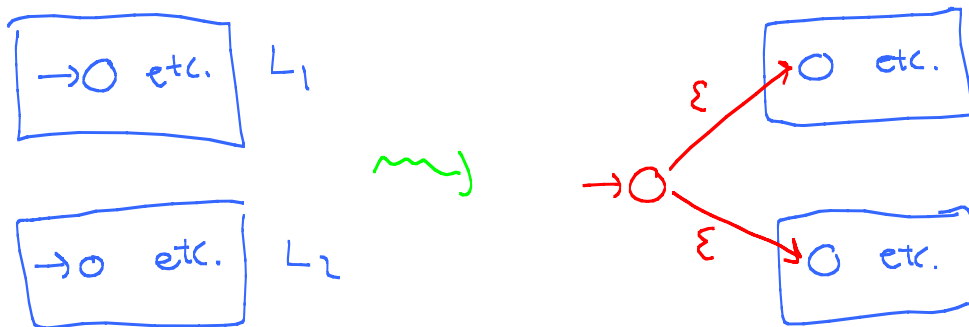


Marks

- [10] 1. Give a **brief** explanation of how to take an NFA accepting a language, L , and form a new NFA accepting L^* . Same question for an NFA for L_1 and one for L_2 , forming a new NFA for $L_1 \cup L_2$. Explain why such constructions can be useful in taking a regular expression for a language and building a DFA accepting it.



Make new initial state with jump (ϵ edge) to old initial, make jumps from old accepting to new state; make new initial accepting



Create new initial state with jumps to each old initial state

This enables one to take a regular expression, built of primitives and $*$, \cup , and form a corresponding NFA, which may then be converted to a DFA.

- [10] 2. Write a two state DFA for the language, L , of words over the alphabet $\Sigma = \{0, 1, \dots, 9\}$ that in decimal represent integers divisible by 5; convert this to a regular expression using the general GNFA procedure. Is the regular expression you get the “smallest” or “simplest” regular expression for L ?

See homework solutions

[8] **3.** Solve either part (a) or part (b), and explain your solution:

(a) Let S be the set of all sets that don't contain themselves, i.e.,

$$S = \{T \mid T \text{ is a set with } T \notin T\}.$$

Show that $S \in S$ leads to a contradiction. Show that $S \notin S$ leads to a contradiction. Do you think there can exist a "reasonable" set theory in which S above exists (and is a set)?

(b) Let $L = \{a^5, a^7\}$. What is the minimum number of states needed in a DFA that recognizes L ? L^* ? Explain your answers, and if you use Myhill-Nerode, give the relevant equivalence classes. [Hint: 23 is the largest integer that cannot be written as non-negative multiples of 5 and 7.]

See homework solutions

- [12] 4. Determine whether or not $L = \{0^n 1^n \mid n \geq 0\}$ is regular and whether or not it is context-free. Same problem for $L = \{0^n 1^n \mid n \text{ is a power of } 10\}$.

(First language done in class and book.) $S \rightarrow 0S1 \mid \epsilon$

shows $L_1 = \{0^n 1^n \mid n \geq 0\}$ is CF. L_1 is not regular,

since if p is the pumping length, we have $0^p 1^p = w_1 w_2 w_3$

with $|w_1 w_2| \leq p$ (so w_2 is all 0's), $w_2 \neq \epsilon$, and

$w_1 w_2^a w_3 \in L_1$ for all a ; but for $a > 1$, $w_1 w_2^a w_3$

has more 0's than 1's.

For $L_2 = \{0^n 1^n \mid n \text{ is power of } 10\}$, same argument

shows L_2 is not regular. L_2 is not CF, since

if w_2 and w_4 are not both 0, then

$$|w_1 w_2 w_3 w_4 w_5| < |w_1 w_2^2 w_3 w_4^2 w_5| \leq 2 |w_1 w_2 w_3 w_4 w_5|;$$

so if $w = w_1 w_2 w_3 w_4 w_5 \in L_2$, then $|w| = 2 \cdot 10^k$ for

some k , $\tilde{w} = w_1 w_2^2 w_3 w_4^2 w_5$ has

$$2 \cdot 10^k < |\tilde{w}| \leq 4 \cdot 10^k < 2 \cdot 10^{k+1}$$

so $\tilde{w} \notin L_2$. So no word of L_2 can be "pumped" in the sense of the Pumping Lemma for CFL's.

The End

Be sure that this examination has 6 pages including this cover

The University of British Columbia

Midterm Examinations - ~~March~~ 2007

October

Computer Science 421/501

Closed book examination

Time: 50 ~~hours~~ ^{minutes}

Name Answer key Signature _____

Student Number _____ Instructor's Name _____

Section Number _____

Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam. A sheet of notes will be provided.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations. !

1		10
2		10
3		8
4		12
Total		40