[5] 1. Write a regular expression for the language over $\{0, 1\}$ of strings with an even number of 0's.

[5] 2. Give a brief explanation of how to take two NFA's and form a new NFA accepting the union of the languages accepted by the two NFA's.

[10] 3. Consider the DFA, $(Q, \Sigma, \delta, q_0, F)$ such that $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1\}$, $F = \{q_0\}$, and $\delta(q_i, j)$ equals q_k where k is $i + j \bmod 3$ (e.g., $\delta(q_2, 0) = q_2$ and $\delta(q_2, 1) = q_0$).

(a) Draw a diagram of the above DFA with the notation given in class and the text.

(b) Find a regular expression for the language accepted by the above DFA using the technique in class and the text of passing to a GNFA and eliminating states one by one.

[10] 4. Let L be the language of words of the form $0^n 1^{2n}$ for some integer $n \geq 1$. Give a CFG describing L, and a pushdown automaton accepting L. Show that L is not regular.

[10] 5. We say that strings x, y are *distinguishable* by a language, L, if there is a string z such that exactly one of xz, yz lies in L.

(a) Let EVEN-EVEN be the set of words in $\{0,1\}^*$ with an even number of 1's and an even number of 0's. Give a set, X, of four strings that are pairwise distinguishable by EVEN-EVEN (i.e., every two strings in X are distinguishable), and explain why X has this property.

(b) Consider a language, L, with a set, X, of p strings that are pairwise distinguishable by L. Explain why there is no DFA recognizing L with fewer than p states.

(c) Let L be the language of strings in $\{0,1\}^*$ whose third last character is a 1 (for example, $000100 \in L$). Give a set, X, of 8 strings that are pairwise distinguishable by L, and explain why X has this property.

The University of British Columbia
Midterm Examination
October 27, 2004

Computer Science 421
Section 101
Instructor: Prof. Friedman

Duration: 50 minutes

(1) Be sure that you have 7 pages in addition to this one.
(2) Put your name below and on the **back** of the other pages. Write it as ⟨**last name**⟩, ⟨**first name**⟩.
(3) In all questions, **you must show work** — i.e. display intermediate results — to get full credit.
(4) You have two blank pages at the end for additional work.

First Name _____ **Last Name** _____

Signature _____ **Student Number** _____

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- Each candidate should be prepared to produce upon request his library/AMS card.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
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  (b) Speaking or communicating with other candidates.
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1.	2.	3.	4.	5.	Total
5	5	10	10	10	40