HOMEWORK 4, CPSC 421/501, FALL 2015

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1. Show that P is closed under union, concatenation, and star. In other words, show that if $L_1, L_2 \in \mathrm{P}$, then $L_1 \cup L_2$, $L_1 \circ L_2$, and L_1^* are in P. [Hint: For the operation "star," you might try dynamic programming.]

2. Write a 3CNF formula for the Boolean formula:

$$f(x_1, \ldots, x_n) = (x_1 \text{ AND } x_2 \text{ AND } \cdots \text{ AND } x_{n-1}) \text{ IMPLIES } x_n$$

whose size is linear in n. [Hint: you may have to introduce some additional variables.]

3. Let

$$\mathrm{SIMPLE-NP} = \{\langle M, w, 1^t\rangle \mid M \text{ is a NTM that accepts } w \text{ on some computation path within time } t\},$$

i.e., the language consisting of a non-deterministic Turing machine, M, an input, w, to M, such that at least one computation path halts within time t and accepts w. Show that the above langauge is NP-complete (from scratch), i.e., show that SIMPLE-NP is in NP, and that any langauge in NP can be reduced to SIMPLE-NP. (Note that the time t is specified in unary, i.e., as a string of t 1's.) Is the NP-completeness of SIMPLE-NP as surprising as that of SAT or SUBSET-SUM? Explain.

The above idea will give us other complete problems in various other classes.

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