CPSC 421/501 Nov 25

Ch 8: Space

8.1 Savitch's Theorem

[Allusions to 8.2,...]

$\mathrm{SPACE}(f(n)) = \{ L \mid L$ can be decided in space $\le C\, f(n) \}$

$\mathrm{NSPACE}(\quad) = \{$ — — —

-- non-determinstic Turing machine

$\dots \}$

$$\mathrm{PSPACE} = \bigcup_k \mathrm{SPACE}(n^k)$$

$$\mathrm{NPSPACE} = \bigcup_k \mathrm{NSPACE}(n^k)$$

Savitch's Thm:

$$\mathrm{NSPACE}(f(n)) \subset \mathrm{SPACE}(f^2(n))$$

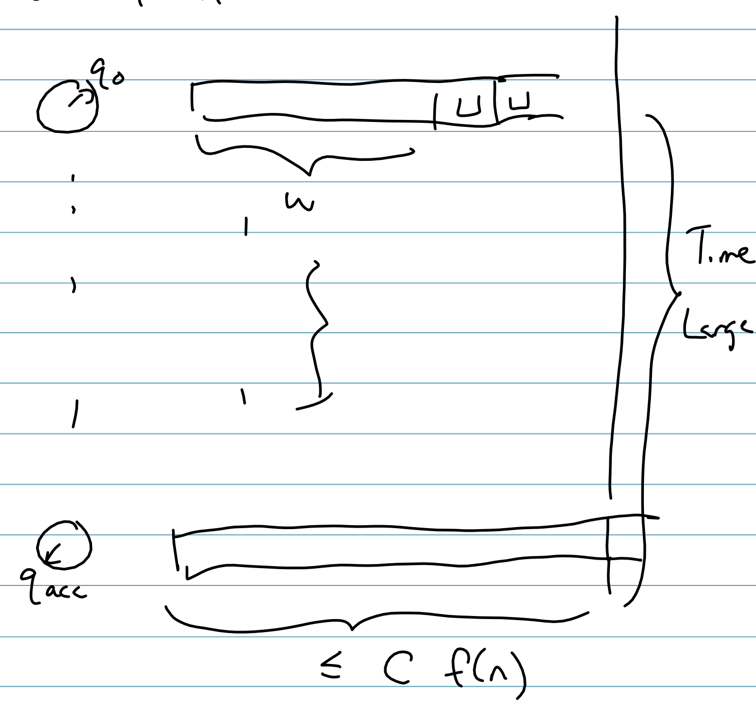
Pf: Let $L \in \mathrm{NSPACE}(f(n))$

Claim

$$L \in \mathrm{TIME}\left(2^{C f(n)}\right)$$

for some C.

We know $\exists$ non-det T.m, M, such that



Config time 1   Initial

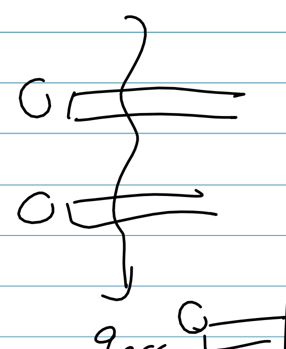
Config time 2

Config time 5

Config time 20

Config time $C f(n)$ q_{acc}

delete



If config time 5 = --- 20

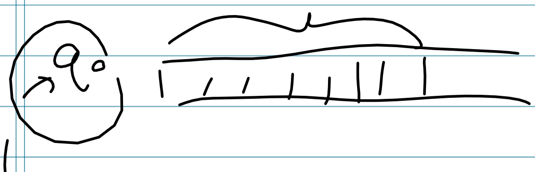
we can delete part between config 5 to config 20

$\leq C_1^{f(n)}$ or $2^{C_2 f(n)}$

Now say # config in a computation reaching q_{acc}

$\leq 2^{C_2 f(n)}$

w

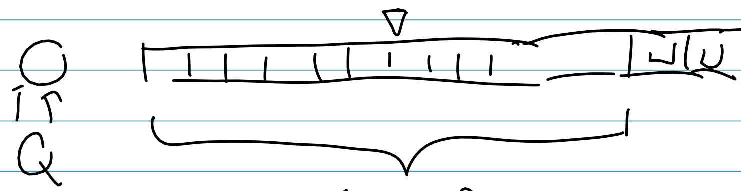


$\leq 2^{C_2 f(n)}$ steps

q_{acc}

If w can be decided and reaches q_{acc}, it can be done without reaching the same configuration twice.

How many config?



$\leq C f(n)$

$\leq |Q| \times |\Gamma|^{C f(n)} \times C f(n)$

states — what written — tape head

First! What is config 1?

What is config $2^{C_2 f(n)}$?

write in space $C f_2(n)$

Second! What is in the middle?

Can we get from config 1 to middle config?

Can we get from middle config to final?

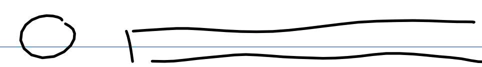
Can we get from config 1 to config $\frac{1}{4} \cdot 2^{C_2 f(n)}$?

Can we get from config $\frac{1}{4} 2^{C_2 f(n)}$ to middle?

Algorithm:

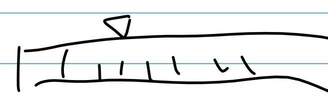
initial only one possibility

middle config



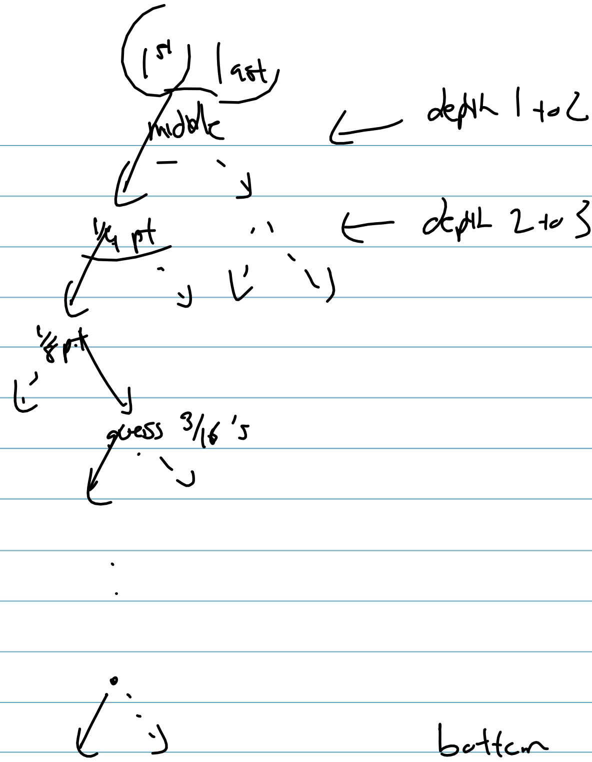
$2^{C_2 f(n)}$ possib

final: q_{acc}



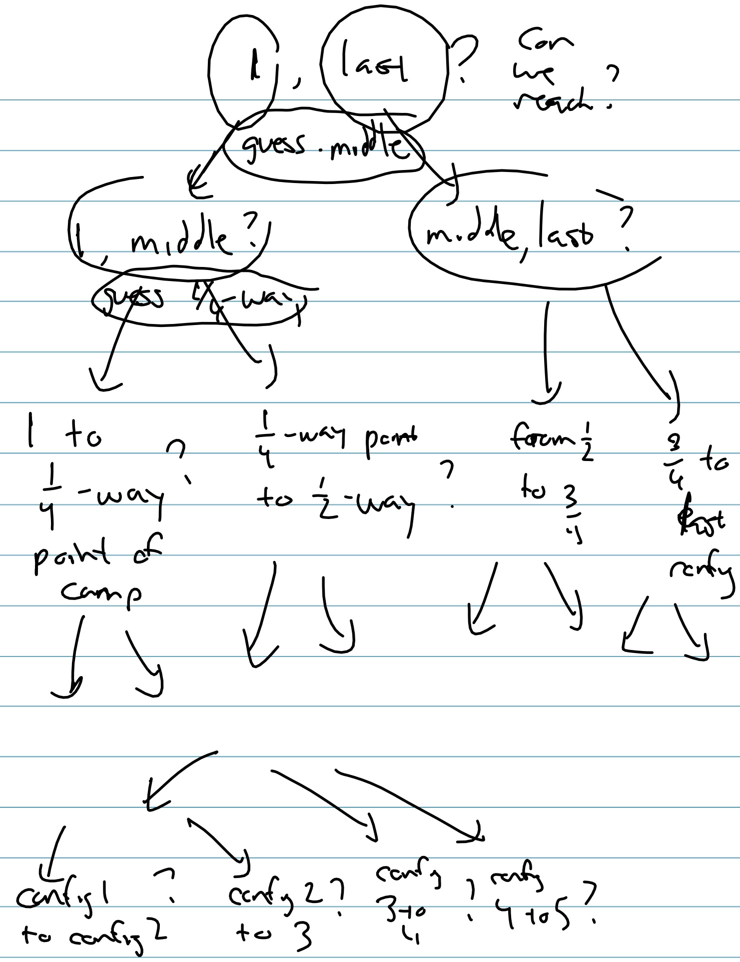
$\leq 2^{C_2 f(n)}$ poss

recurse



depth d such that

$$\frac{1}{2^d}(\#\text{time}) \le 1$$



Need to remember where you are in tree:

$$\underbrace{L/R,\ L/R,\ \ldots,\ L/R}_{d}$$

Space: order(d)

=

Total Space:

$(d \text{ or } d+1)$ configs $\cdot$ $(C_2 f(n) + \text{small})$ per config

$+$ order(d) where are we in tree $\le$ order$(f(n)^2)$

If w accepted, it is accepted by a path of time $\le 2^{C_2 f(n)}$

So depth need:

$$\frac{1}{2^d}\, 2^{C_2(f(n))} \le 1$$

i.e. $d = C_2 f(n)$

To write down: 1st config, last, middle, any config: space $C_2 f(n)$ per config