CPSC 421/501 Nov 23

Goal 1: $\exists$ oracle B s.t.

$$P^B = NP^B$$

(also $\exists$ oracle A s.t. $P^A \neq NP^A$)

In Ch 8 & Ch 9

Ch 8: SPACE

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Goal 2: Review a lot of what we have done...

TIME, space, reductions, ...

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Leftover: Sample Exam Problems

Ch 8: space:

If $f: \mathbb{N} \to \mathbb{N}$, we define

$SPACE(f(n)) = \{ L \mid$ decidable in space $O(f(n)) \}$

Decidable: $\exists$ Turing machine, M, constant, C, s.t.

(1) $w \in \Sigma_L^* = \Sigma_{TM}^*$ is in $L \iff M$ on input w reaches q_{acc} in ~~time~~ using at most $C\, f(|w|)$ tape cells

2 possibilities for w to be **rejected**:

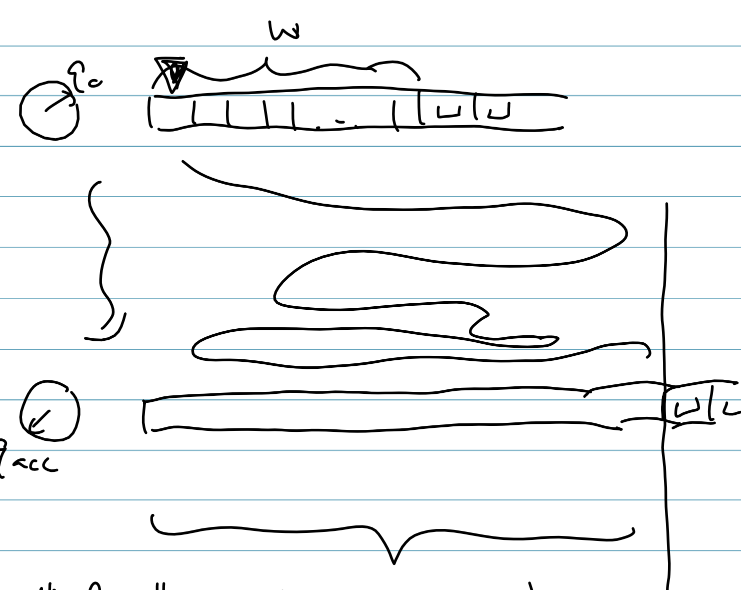
① using $O(f(n))$ cells the T.m. reaches q_{rej}

② T.m. either

- uses more that $C\, f(|w|)$ tape cells
- reaches q_{rej}

→ **Decide:** stops within the resources

Recognize: does not accept within the resources



\# of cells reached $\leq C\, f(|w|)$

(the rightmost cell the tape head reaches)

$\mathrm{NSPACE}(f(n)) = \{ L \mid$
L - - - non-deterministic Turing machine - - in space $\leq C\, f(n) \}$

for $f(n) = n^k$ $(k=1,2,3,\ldots)$
talk about <u>deciding</u>, i.e.
insist $\left\{\begin{array}{l} \text{Turing machine} \\ \text{non-det } '' \quad '' \end{array}\right\}$
always halts, in either q_{acc} or q_{rej}, in space $\leq C\, f(n) = C\, n^k$.

Say:

$$\mathrm{PSPACE} = \bigcup_{k=1,2,\ldots} \mathrm{SPACE}(n^k)$$

As long as $f(n)$ can be computed in <u>space</u> $O(f(n))$, ~~they~~ Turing machine can be modified to make sure it stops after ~~time~~ ~~or~~ <u>space</u> $\leq C\, f(n)$

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There are $f(n)$ that cannot be computed! functions that grow very fast

Claim:

$\left\{\begin{array}{l} \forall x_1 \exists x_2 \forall x_3 \exists x_4 \ldots \exists x_{2k} \\ \text{s.t. formula}(x_1, x_2, \ldots, x_{2k}) \ldots \end{array}\right\}$

is this true? Can be done in PSPACE

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SAT: $\exists x_1, x_2, \ldots, x_n$ s.t. $f(x_1, \ldots, x_n)$ is true?

Quantified SAT:
$\forall x_1 \exists x_2 \forall x_3 \exists x_4 \ldots$
Chess, games

Thm: $\mathrm{N}\cancel{\mathrm{P}}\mathrm{SPACE} = \mathrm{PSPACE}$

<u>Savitch's Thm:</u>

$$\mathrm{NSPACE}(n^k) \subset \mathrm{SPACE}(n^{2k})$$

$$\mathrm{NSPACE}(f(n)) \subset \mathrm{SPACE}(f^2(n))$$

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Example:

Boolean formula

$(x_1 \vee x_2 \vee \neg x_3) \wedge (\quad) \ldots$

can decide this in non-det poly time.

Boolean game:

$\exists x_1 \forall x_2 \exists x_3 \dots \exists x_{2k}$

s.t. $f(x_1, \dots, x_{2k}) = \text{True}$

So input: $\langle \underbrace{f(x_1, \dots, x_{2k})}, k \rangle$

$\underbrace{((x_1 \text{ and } x_2) \text{ or } \dots), k}_{\text{input } w}$

Quantified-SAT-type-problem:

$\langle \underbrace{\quad}_{\text{formula}}, k \rangle$

TIME $\leq$ order(size(formula))

SPACE $\leq$ # variables.

I win a game if

$\exists$ a move for me

$\forall$ of my opponents moves

$\exists$ a 2nd move for me

$\forall$ of my opp 2nd moves

$\exists$ . . .

$\forall$

$\vdots$

$=$

$\exists x_1, x_2, x_3 \, \forall x_4 x_5 x_6 \, \exists \dots$

Fact 1:

$\text{SPACE}(n^k) \subseteq$

$\text{NSPACE}(n^k) \subseteq$

$\bigcup_c \text{TIME}(c^{n^k})$