CPSC 421/501 Nov. 20

- One more NP-reduction
- SPACE

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Any $L \in NP$ or any non-det TM can be expressed as a problem in SAT. Big idea.

$\rightsquigarrow$ 3SAT is also NP-complete (relatively easy modification)

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SUBSET-SUM is NP-complete

Big idea

—

Now PARTITION...

PARTITION

$= \{ \langle n_1, \ldots, n_k \rangle \text{ s.t. } \exists\, I \subset \{1, \ldots, k\}$ for which $\sum_{i \in I} n_i = \sum_{i \notin I} n_i \}$

e.g.

$(2, 3, 5, 7, 9, 12) \in$ PARTITION

since

$2 + 5 + 3 + 9 = 7 + 12$

but

$(2, 3, 5, 7, 9, 11, 13, 15, 1000)$

is not

1000 + $\neq$

never equal

Claim: PARTITION is NP-complete

Proof:

① PARTITION $\in NP$

② If $L \in NP$, then

$L \leq$ PARTITION

Why? $L \leq$ SUBSET-SUM

Why?

We know $L \leq$ SAT $(\leq)$ 3SAT

$\leq$ SUBSET-SUM

$\leq$ ← poly time

It suffices to show

SUBSET-SUM $\underset{\text{poly time}}{\leq}$ PARTITION

i.e.

you have SUBSET-SUM problem:

$w = (2, 7, 1, 20, 15, 35 ; 40)$

subsequence whose sum is 40

Construct

$w \mapsto f(w)$ s.t.

$w \in$ SUBSET-SUM $\iff f(w) \in$ PARTITION

First attempt

$(1, 2, 3, 4, 5, 6, 10, 11)$

$1+2+3+4 = 10$

$5+6 = 11$

$\underbrace{1+2+3+4+5+6 = 10+11}_{\text{partition}}$

=

$(1, 4, 7 ; 10) \notin$ SUBSET SUM

$\Sigma = 12$

$1, 4, 7, 10, 2$

orig target   leftover

$1+4+7 = 10+2$

$2, 7, 1, 20, 15, 35 ; 40$

PARTITION problem

$n_1, \ldots, n_k$:

$$\sum_{i \in I} n_i = \sum_{i \notin I} n_i$$

=

$1, 2, 3, 4, 5, 6 ; 10$ } SUBSET-Sum

$1+2+3+4 = 10$

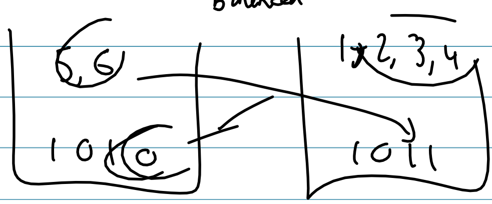
(leftover $5, 6 = 11$)

=

$4+6 = 10$

(leftover $1, 2, 3, 5 = 11$)

balanced



$1, 2, 3, 4, 5, 6, 1010, 1011$

target +1000   leftover +1000

$1+2+3+4 = 10$

=

$w = \langle n_1, \ldots, n_k ; m \rangle \leftarrow$ target

If: $\sum_I n_i = m$

$1, 4, 7, 10, 2$

$1, 4, 7, 1010, 1002$

very big

1010   1002

=

$1, 2, 3, 4, 5, 6 ; 10$

leftover $= 11$

these balance:

$$\sum_{j \notin J} n_j + m = \sum_{j \in J} n_j + \ell$$

so

$$\sum_{j \in J} n_j = m - \ell - \sum_{j \notin J} n_j$$

$$= m - \ell + \left(m + \ell - \sum_{j \in J} n_j\right)$$

$$\left(\text{but } \sum_{j \in J} n_j + \sum_{j \notin J} n_j = m + \ell\right)$$

$$2 \sum_{j \in J} n_j = 2(\cancel{m - \ell})$$

$$= 2m$$

$\Rightarrow$ $n_1, \dots, n_k, m + 10^{|w|}, \ell + 10^{|w|}$

where

$$\ell = n_1 + \dots + n_k - m$$

then

$\{n_i \mid i \notin I\}$ $\boxed{m + 10^{|w|}}$ $\qquad$ $\{n_i \mid i \in I\}$ $\boxed{\ell + 10^{|w|}}$

then $\in$ PARTITION.

Also if $\{n_j \mid j \notin J\}$ $\boxed{m + 10^{|w|}}$ $\qquad$ $\{n_j \mid j \in J\}$ $\boxed{\ell + 10^{|w|}}$

$w \in$ SUB.. $\Rightarrow f(w) \in$ PART..

($w \notin$ SUB.. $\Rightarrow f(w) \notin$ PART

$f(w) \in$ PART $\Rightarrow w \in$ SUBS.)

more often argue

$\langle 1, 2, 3, 4, 5, 6 \,;\, 10 \rangle = w$

$\langle 1, 2, 3, 4, 5, 6, 10^{|w|} + m, 10^{|w|} + \ell \rangle$

$f(w)$

$$|f(w)| \leq |w| + 2|w|$$

time: time it takes to add $1+2+3+4+5+6$, subtract 10

$\rightarrow$ leftover $\rightsquigarrow$