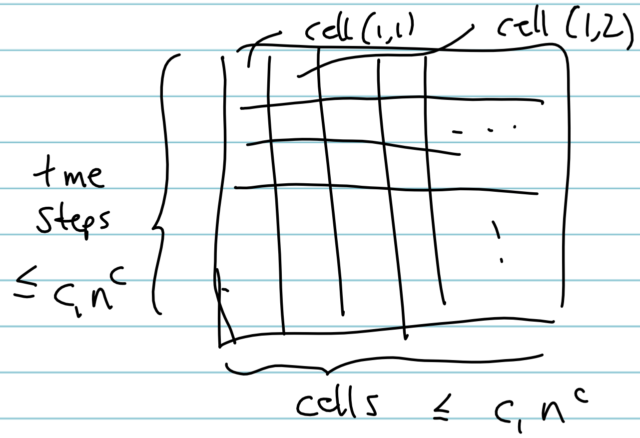
Nov 13, CPSC 421/501

Ch 7:

Last time:

Cook-Levin Thm: If $L = NTIME(n^c)$ (c integer, > 0) then there is a formula $f(w)$, for each $w \in \Sigma_L^*$ such that

$\boxed{1}$ Size $f(w) \leq C|w|^{2c}$



L there is a NTM, M, decides L in time $\leq c_1 n^c$

formula: blah AND blah AND blah AND --

① step 1, w is on input tape, head cell 1, state q_0

② reach q_{acc}

③ each step transition is permitted by TM

Formally:

Def: L to be NP-complete ("a hardest problem in NP") if

① $L \in NP$

② $\forall L' \in NP$; there is reduction $L' \underset{\text{poly time}}{\leq} L$

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E.g. SAT ~~is~~ NP-complete

Now $\left\{ \begin{array}{l} \text{3SAT, SUBSET-SUM,} \\ \text{3COLOR, INDEPENDENT-SET} \\ \text{CLIQUE, ...} \end{array} \right\}$ to be NP-complete

– 3SAT is usual starting point...

– Why it makes things easy, e.g. SUBSET-SUM

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3CNF (3 Conjunctive Normal Form):

$(x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_1 \vee x_5 \vee x_4)$

$\wedge (x_2 \vee \neg x_1 \vee x_3) \wedge \ldots$

$\wedge$ AND $\qquad \vee$ OR

Really want $f(w)$ to be 3CNF...

$\boxed{2}$ f is computable in poly time

$\boxed{3}$ $\forall\, w \in \Sigma_L^*$,

$$w \in L \iff f(w) \in SAT$$

Hence $L \in NP$ $\exists\, f : \Sigma_L^* \to \Sigma_{SAT}^*$

s.t. - f poly time computable.

- $w \in \Sigma_L^*$. $w \in L \iff f(w) \in SAT$

Cor: $SAT \in P \iff NP = P$

Pf: $w \in L$, $f(w)$ computable in poly time,

$|f(w)| \leq C|w|^{2c}$, $SAT \in TIME(n^k)$

and $L \in NTIME(n^c)$

$\Rightarrow |f(w)| \leq C_1 |w|^{2c}$, SAT runs in

time $\leq (C_1 |w|^{2c})^k = C_1^k |w|^{2ck}$

So $SAT \in P$, $NP = P$

If $SAT \notin P$, $NP \neq P$

At some point in Cook-Levin

we have $(x_{..} \text{ OR } x_{..} \text{ OR } x_{..} \text{ OR } x_{..})$

i.e.

→ if $(x_{..} \text{ and } x_{..} \text{ and } x_{..})$ then $x_{..}$

→ if $\underbrace{\qquad\qquad}_{p}$ then q

→ $(\neg p \text{ OR } q)$

→ $(\neg x_{..} \text{ OR } \neg x_{..} \text{ OR } \neg x_{..})$ or $x_{..}$

So $(x_1 \text{ OR } x_2 \text{ OR } x_3 \text{ OR } x_4)$

↳ 3CNF

we said

but x_1 OR x_2 or x_3 or x_4

$\cong (x_1 \text{ OR } x_2 \text{ OR } y) \text{ AND } (\neg y \text{ OR } x_3 \text{ OR } x_4)$

(adding y)

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Given 3CNF ⇝ SUBSET-SUM

$\underbrace{n_1, \ldots, n_k}, t$

? if there subcollection that add to t

Midterms !

- simple T. m (8 pts) : 8/33 is a pass
- problem 2: $|S| < |Power(S)|$

- Problem 3!

Part (d) TM runs in $n+10$

a, b can run $\leq a n^{b}$

$b = 2$ 2/4

$a =$ anything 2/4

} 4/4 $O(n^2)$

explain why

8 → 50

Median 19 → 72

31 → 100

(A-K
L-S
T-Z)