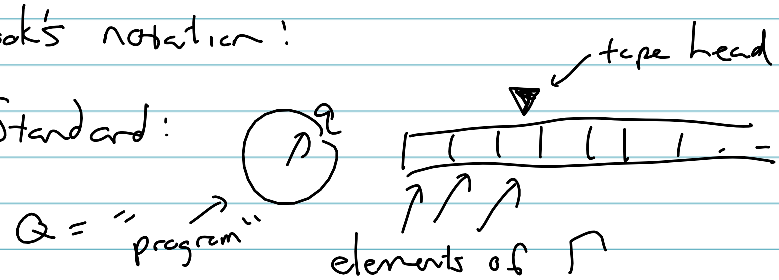
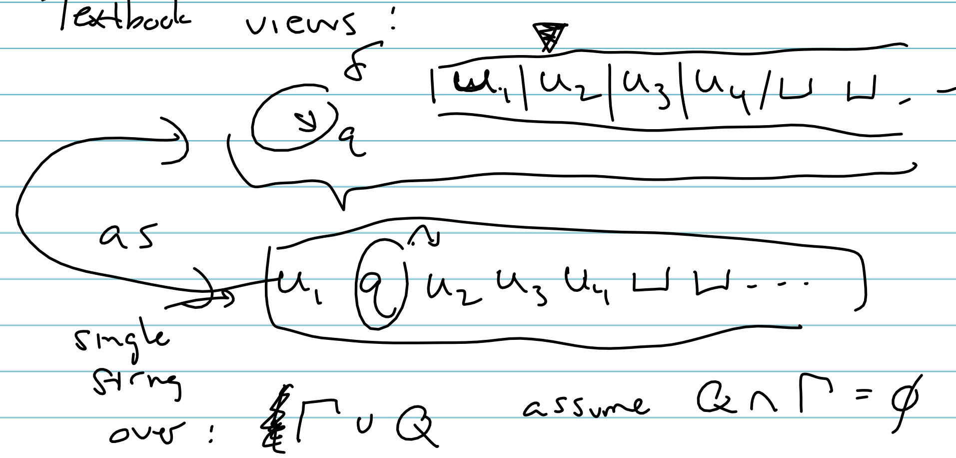
CPSC 421/501 Nov 9

Book's notation:

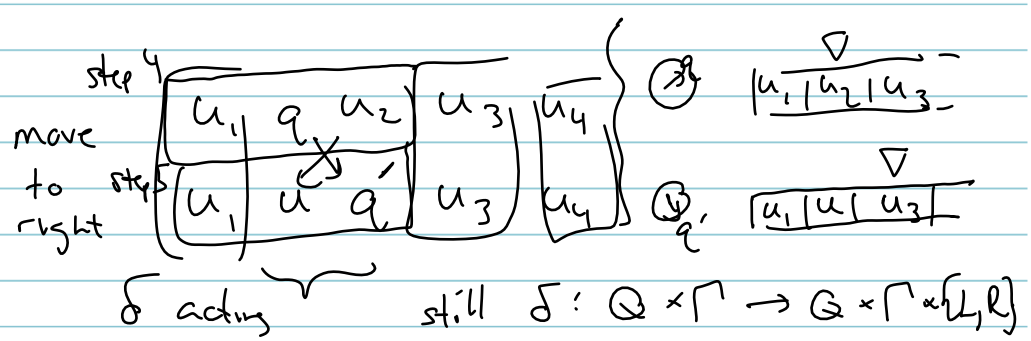
Standard:



Textbook views:



assume $Q \cap \Gamma = \emptyset$



still $\delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

Cook-Levin Theorem: If L ~~accepted~~ decided by NTM in time $O(n^c)$, $L \leq_p$ SAT, 3SAT

First: Express ①, ②, ③ as <u>Boolean formulas</u>

<u>involving</u> $x_{i,j,k}$

Second: Make sure we can write ①, ②, ③

in poly time: NTM M fixed, $\Sigma = \Sigma_M = \Sigma_L$

<u>L decided by M</u>: $f: \Sigma_L^* \longrightarrow \Sigma_{SAT}^*$

must be poly time in length of word

<u>Third</u>: See that this

formula can be written as

( ) and ( ) and ( ) and ... and ( )

or of 3 variables or negations

3CNF

3 conjunctive normal form

=

① Initial condition ③ final ② expensive: step $i \to i+1$

is allowed
$$\begin{array}{c|c|c} & \downarrow j & \\ \hline \gamma_1 & q & \gamma_2 \\ \hline \gamma_1 & \gamma_2' & q' \end{array}$$
if $(\gamma_2', q', R) \in \delta(q, \gamma_2)$

$\delta(q, \gamma_2)$ has (q^1, γ^1, L), (q^2, γ^2, R), ...

if $(\gamma_2', q', R) \in \delta(q, \gamma_2)$ and step i we see

γ_1 at cell $j-1$, q cell j, γ_2 cell $j+1$

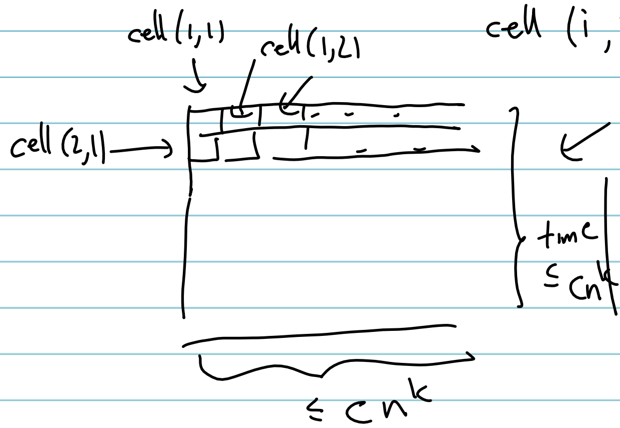
... then γ_1 at step $i+1$, cell $j-1$, γ_2' at step $i+1$ cell j, ...

Where $\le_p$ a reduction:

$\Sigma_L \xrightarrow{f} \Sigma_{SAT}$ where f can be computed in poly time.

In particular: if $SAT \in P$, then $NP = P$.

<u>Proof</u>: $X_{i,j,k}$ = we see symbol $\# k$ from $Q \cup \Gamma$ in cell (i,j)



this is how NTM $Q \cup \Gamma$ works over time $\le cn^k$

cell(i,j), $i = 1, \ldots, cn^k$
$j = 1, \ldots, cn^k$

We want:

AND (① At time = step $\# 1$, cell(1,1), cell(1,2), ... looks like $| q_0 | w_1 | \ldots | w_n | \sqcup | \sqcup | \ldots$

AND (② By time cn^k, we reach q_{acc}

③ From step i to step $i+1$, the transition is legal for δ

(r_2', q', R) is the 10^{th} element of $\delta(q, r_2)$

and cell$(i, j-1) = r_1$, cell$(i, j) = q_1$ ...

then r_1 cell$(i+1, j-1)$, q' at cell$(i+1, j)$ ...

i.e. = if (blah$_1$ and blah$_2$ and blah$_3$) then cell$(i+1, j-1) = r_1$

AND - - - - - - - - then cell$(i+1, j) = q'$

AND

AND( ) AND ( ) AND ( ) ...

if (p_1 AND p_2 AND p_3) then q

=

(if p then q) $\Leftrightarrow$ ($\neg p$ OR q)

(if (p_1 AND p_2 AND p_3) then q)

$\Leftrightarrow$ $\neg(p_1$ AND p_2 AND $p_3)$ OR q

$\Leftrightarrow$ $(\neg p_1$ OR $\neg p_2$ OR $\neg p_3)$ OR q

$\rightsquigarrow$ Boolean alg $\rightsquigarrow$ (1) AND (2) AND (3) $|Q \cup \Gamma|$

can be written as ( ) AND ( ) AND ...

each ( ) is a bunch of OR's of $X_{i,j,k}$

$1..cn^k$ $\quad$ $1..cn^k$